A MANUAL

OF

PHYSICAL MEASUREMENTS

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PREFACE.

This manual has been prepared to meet the needs of students beginning work in the Physical Laboratory of the University of Michigan. Such a book must inevitably possess a certain local coloring peculiar to the conditions it has been designed to meet. A manual equally suited to all laboratories, has not been and probably will not be written. Each laboratory reflects in greater or less degree the individual trend of the man who stands at its head, and its exercises and methods are the result of an extended process of adaptation and assimilation. Hence it happens that one laboratory is largely devoted to the study of the phenomena of light, another to those of electricity, and a third to those of elasticity, heat, or electro-chemistry, as the case may be. The moral of all this is, that the practices and traditions of each laboratory are best conserved by a text representative of its own methods, and if no better reason should be found, perhaps this may serve to explain the appearance of this, another laboratory manual.

The exercises herein described embody the work required of students in Physics and in Engineering in

their first course in Physical Laboratory Practice. Such a course is expected to occupy three laboratory periods of two hours each for one semester, and embraces some thirty-six to forty of the exercises in this manual. Owing to the diversity of the work prescribed in the various courses in Engineering, no one student is expected to complete all the exercises in this book in a single semester.

In accordance with the practice in the University of Michigan, it is expected that the laboratory work shall be supplemented by lectures upon the theory of the exercises, and recitations upon the work actually done and the results obtained. In this way it is believed that the student is brought to a clearer understanding of the significance of the exercise and of the accuracy attainable under given conditions. To this end the exercises are numbered consecutively throughout the text, and those under any specific subject are preceded by sufficient theory to render the formulæ and methods clear to persons familiar with the fundamental principles of Physics as set forth in any standard textbook.

Being designed for beginners in the Physical Laboratory, this manual makes no claim to completeness, either in subject matter or in exposition. The aim has been to furnish a coherent and logical series of graded exercises in Physical Measurement, such as will best furnish an introduction to Practical Physics, and at the same time afford opportunity for developing ability in recording and interpreting observations, and skill in the manipulation of delicate and sensitive apparatus.

For convenience of reference a series of tables of the more important physicai constants, of squares, cubes, square roots and reciprocals, of the logarithms of numbers,

and the trigonometric functions have been added. A thorough drill in the use of logarithmic tables in the computation of results, should form a feature of any successful course in Laboratory Practice. To this end an orderly method of procedure in such computation has at all times been insisted upon.

The authors have drawn freely from many standard works on Practical Physics, notably from those of Kohlrausch, and Stewart and Gee in General Physics, and from Carhart and Patterson's Electrical Measurements.

In conclusion we wish to thank our colleagues, Professors Carhart and Patterson, for helpful suggestions and criticisms during the preparation of the work.

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INTRODUCTION.

The benefits to be derived by the student from work in the physical laboratory are two-fold. In the first place he is to become acquainted with delicate instruments, to be trained to make systematic, accurate and independent observations, and to compute, from the data so obtained, the values of many of the more important physical constants. Secondly, he is to make a searching review of the fundamental principles of the science, and to be brought to a more lively realization of the meaning and importance of formulae and laws deduced in the text-books upon general physics. To this end it is urgently advised that the student familiarize himself with all the details of the theory of the experiment, and *be able to sketch from memory* the apparatus to be employed before beginning any experiment. An attempt to follow directions but dimly understood, and to manipulate apparatus whose construction and purpose are alike unknown, can only result in loss of time, and laboratory work under such circumstances is practically worthless as a means of discipline.

Instruments. The instruments used in the physical laboratory are usually of delicate construction, many of them costly and liable to injury from rough or careless usage. It is of the highest importance that all apparatus should be handled with care, and returned to its proper place after use. If any piece is found to be out of adjustment or in need of repair, report the fact before beginning work. If any screws or other parts of an instrument, do not move readily, do not apply force but report the matter to the instructor in charge. The ability to use delicate apparatus without injuring or destroying it, is an important part of a liberal education.

Record of Observations. All observations, data and necessary formulae, such as the time and place of the exercise, the specific instruments used, and objects measured etc., are to be recorded in a note book provided for that purpose. Such a book is to have *fixed leaves*, and to be made of paper suitable for writing with ink. The record is to be made *at the time of the experiment.* It must contain the detailed information necessary, must be clearly written and arranged in a neat, methodical manner, so that one familiar with the experiment may readily comprehend what has been done. It should be sufficiently specific to be intelligible after the circumstances of the experiment are entirely forgotten.

A suitable form of record has been appended to each exercise in the manual, and the student will do well to follow these, at least until he is able to arrange the material for himself. Tabulation of results in columns adds much to the neatness of a note book and materially assists in the detection of errors, either of record or of observation. It is suggested that the note books should be casually

inspected by the instructor, in his rounds in the laboratory at each exercise, if possible.

The computation of results should generally be done at home, and the complete experiment recorded in a separate note book to be handed to the instructor as the final report upon the work of the course. In this book the date, number, name and object of the exercise should precede the record of observations and the computed results. This material together with any necessary explanatory memoranda should be *written on the right hand page*, the left hand page being reserved for sketches of apparatus, and the derivation of necessary formulae. A separate page should always be begun for each new experiment.

Graphical Methods. It is frequently of interest to verify a law, stating the relation that is known to exist between two quantities, or to detect and determine such a relation where it is not known. In all such cases the results obtained by observation are most clearly presented to the eye when plotted as a curve. In the application of the method it is customary to plot values of the independent variable as abscissae and those of the dependent variable as ordinates. In all cases where the phenomenon under investigation is continuous, a smooth curve sketched through the points obtained, may be assumed to represent the facts better than any individual observation. The graphical method has the additional advantage that an accidental error is at once made evident by the fact that *the point so obtained departs markedly from the curve.*

The curve most readily plotted and tested is *the straight line*, and it is customary so to transform the assumed relation as to render the plotting of a straight line practicable.

For example, suppose it were desired to investigate the relation between the time of vibration of a pendulum and its length. If we assume that this relation may be expressed by an algebraic function of the form

$$T = a\,l^m$$

we may determine the constants a and m from a series of observations. Passing to logarithms we have

$$\log T = m \log l + \log a.$$

This is clearly of the form

$$y = m\,x + c$$

and is therefore the equation of a straight line. If now we plot values of log l as abscissae and the corresponding values of log T as ordinates, we may decide at once whether such a relation as we have assumed exists, and we may obtain values of a and m directly from the curve.

It is not necessary that the same numerical value should be assigned to a scale division on the horizontal and vertical axes. In general it is best to make the value of a scale division correspond, as nearly as possible, to the least quantity which we can measure. If this be impossible, such values should be chosen for a scale division on each axis as will cause the curve most nearly to fill the page with the observations to be plotted. It is only in case we wish to determine from the curve a quantity which is represented by the tangent of an angle, i. e., which represents the ratio between the coordinates, that it is necessary to assign to them their proper relative values.

A table of the data from which the curve has been plotted *should in all cases* accompany the curve, and *the values assigned to a scale division* on the horizontal and vertical axes *must be clearly stated upon these axes.* In practice it is well to prick with a needle the exact position of each

point on the curve and then draw round each point a small circle in colored ink. All curves should be plotted upon special cross-section paper, drawn in ink, and the points clearly marked as indicated above. In case special accuracy is desired it is well to use paper printed from engraved plates. The curves are to be inserted in the note book *after the record of the experiment.* This is most readily done by cutting away about two-thirds of a sheet lengthwise, and pasting the stub to the back of the cross-section paper.

The following data may be used as exercises in the plotting of curves. In case any set of data does *not* represent *continuous phenomena* how should the curve be drawn?

I. POPULATION OF THE UNITED STATES.

Year	Population	Year	Population
1790	3 929 214	1850	23 191 876
1800	5 308 483	1860	31 443 321
1810	7 239 881	1870	38 558 371
1820	9 633 822	1880	50 155 783
1830	12 866 020	1890	62 622 250
1840	17 069 453	1900	76 303 387

II. ATTENDANCE AT THE UNIVERSITY OF MICHIGAN.

Collegiate Year.	No. of Students.	Collegiate Year.	No. of Students.
1881-82	1534	1891-92	2692
1882-83	1440	1892-93	2778
1883-84	1377	1893-94	2659
1884-85	1295	1894-95	2864
1885-86	1401	1895-96	3014
1886-87	1572	1896-97	2975
1887-88	1667	1897-98	3223
1888-89	1882	1898-99	3192
1889-90	2153	1899-1900	3441
1890-91	2420	1900-1901	3712

III. HYSTERESIS CURVE FOR SWEDISH IRON.

The values are given for half a cycle only. To obtain the complete curve, reverse the signs of both columns.

Strength of field H	Magnetic Induction. B
0	— 8300
1.5	— 5816
2.7	0
3.7	+ 3041
5.0	+ 5284
6.6	+ 7037
8.8	+ 8658
11.3	+ 9923
16.0	+ 11389
25.2	+ 12898
36.2	+ 13808
45.9	+ 14430
62.4	+ 15074
45.0	+ 14652
21.1	+ 13653
6.6	+ 11766
4.6	+ 11122
0	+ 8300

Errors of Observation. In any series of measurements of the same physical quantity, we find that the results differ slightly from one another, owing to imperfections of the instrument or errors in making the readings. These errors are not to be confounded with mistakes in calculation or errors in recording observations. These must of course be rejected. First to be considered are the accidental errors of observation which, if the observations have been made without any bias, or preconceived idea as to what the value *"ought to be,"* are as likely to be positive as often as negative and may, in the long run, be considered as having little influence upon the mean result.

Obviously the influence of such errors is diminished by making the number of observations as large as possible and taking the arithmetical mean. This mean value will probably be more nearly correct than any one of the individual observations.

It is not, however, at all times convenient to multiply the number of observations, neither is it at all times necessary. If a set of readings differ but little among themselves, it is clear that little will be gained by increasing the number of such observations. On the other hand if the individual readings differ markedly among themselves, a much larger number must be taken if the average reading is to be considered as trustworthy.

What is more to the purpose is to determine if possible the limit of error of the result, or the *probable accuracy* of the average. This is done by computing what is known as the "probable error" of the mean result. The computation is made by writing the readings in a vertical column, and placing opposite each reading the difference between it and the mean, making it plus or minus, according as the reading is greater or less than the mean. This difference is termed the "residual" for the observation in question. It is shown in the theory of least squares that the probable error of the mean of n observations, is

$$E = 0.6745 \sqrt{\frac{s}{n(n-1)}}$$

where s is the sum of the squares of the residuals of the n observations. If e be the probable error of a set of readings, a the average, and x the true value of the reading sought, then

$$a + e > x > a - e$$

which means that the true value of x lies between the

mean *plus* the probable error and the mean *minus* the probable error.

In addition to accidental errors of observation, there are to be considered the errors arising from faults in the instrument or in the method of observation. These are classed as *systematic errors* and are neither to be eliminated nor estimated by computation. They cannot be removed entirely, but may be minimized by repeated measurements with different instruments in the hands of different observers. Much thought should therefore be expended upon devising correct methods of observation, and means for avoiding systematic errors, since upon these the accuracy of the result must finally depend. Students are to be urged to use judgment in measurements and warned against excessive care in guarding against minute mistakes, while errors of the grossest kind are liable to be made in the process. It frequently occurs that a student in measuring the length of a wire will expend much time in determining the length to hundreths of a millimeter, and yet make an error of a centimeter in the result.

Influence of Errors upon the Result. It is frequently necessary to compute a result from one or more quantities obtained by observation. In such cases it is of interest to determine the influence of errors in the observed quantities upon the computed result. If X be the value sought, and x the value of the quantity observed, then X is some function of x. If e be the error in x due to all causes, and E the error in X consequent upon e, then

$$X + E = f(x + e)$$

It may be shown that the total error is approximately

$$E = e \frac{dX}{dx}.$$

From a consideration of the applications of this formula valuable suggestions as to methods of measurement are often obtained, whereby the percentage of error may be much reduced. Thus let it be required to determine the conditions most conducive to accuracy in the measurement of an electrical resistance by the slide wire bridge. The expression for the resistance measured by means of a slide wire bridge is

$$r = R \frac{a}{c - a}$$

where R is the known resistance in ohms, a the reading on the bridge wire, c scale divisions in length. In this case the expression for E becomes

$$E = e \frac{d\,r}{d\,a} = e\,R \frac{c}{(c - a)^2}$$

and the relative error

$$\frac{E}{X} = \frac{E}{r} = e \frac{c}{a\,(c - a)}$$

This expression will be a minimum for a maximum value of the denominator, a (c − a). But a (c − a) is a maximum when a = c − a, or 2 a = c; that is, the adjustment should be such as to bring the reading to the middle of the scale.

Interpolation. It is frequently desirable to evaluate a physical quantity beyond the limit of the subdivisions of the instrument at our disposal. Thus let it be required to weigh a body to 0.1 mg., while the smallest weight in the box of weights is 1 mg; or let it be required to determine the resistance of a piece of wire to 0.1 ohm, by the method of substitution when the known resistance is subdivided to ohms only. In such cases the method of interpolation is applied. Thus let x be the observed quantity corresponding to the unknown quantity y. We

are to select two quantities X and x, in the neighborhood of x_c such that $X > x_c > x$. Let the corresponding values of y_c be Y and y. Then if these values lie near enough together so that we can assume that $Y - y$ is proportional to $X - x$, we find

$$y_c = y + (x_c - x)\,\frac{Y - y}{X - x}\,.$$

Hints on Computation. The following suggestions regarding the computation of results will be found useful.

(*a*) *Taking the Mean.* In taking the mean of a set of readings, it is necessary to average only those parts of the various readings which *differ* among *themselves.* Thus if ten readings have the first three figures 264, and differ only *in the tenths* it is clearly unnecessary to average more than the tenths.

(*b*) *Significant Figures.* The student should avoid carrying a result out to a large number of decimal places, far beyond the point where the figures have any significance whatever. Thus if six readings of the barometer be 743.3, 743.2, 743.3, 743.1, 743.2, 743.3, the mean is 743.233 mm., of which but *four* figures are significant and the tenths are uncertain since they cannot be read every time alike. If this reading be corrected for temperature the result should likewise be given to *tenths of a millimeter but no farther* since nothing is known beyond that. In general the result should be carried to so many figures that the last, owing to errors makes no pretension to accuracy, while the next to the last may be regarded as reasonably accurate.

(*c*) *Approximations.* In many cases where it is necessary to compute results from values, some of which are very small in comparison to others, the labor may be greatly reduced by approximation formulae. Some of the

more useful are given below, where a, b, c and d are to be regarded as very small in comparison to unity.

$$(1 \pm d)^m = 1 \pm m\,d$$

$$\sqrt{1 \pm d} = 1 \pm \tfrac{1}{2} d$$

$$\frac{1}{\sqrt{1 \pm d}} = 1 \mp \tfrac{1}{2} d$$

$$(1 \pm a)(1 \pm b)(1 \pm c) \cdots = 1 \pm a \pm b \pm c \cdots$$

(*d*) *Supplementary Tables.* At the end of the book will be found a series of tables of mathematical and physical constants. The student will find it of advantage to consult these freely in the course of his work. While the values of the physical constants contained in these tables have been selected from reliable sources, the student is warned against the error of assuming that a value obtained in the laboratory is necessarily wrong, because it differs slightly from that given in the table.

CHAPTER I.

FUNDAMENTAL MEASUREMENTS.

Since most physical quantities may be expressed either directly or indirectly in terms of the fundamental units of mass, length and time, the first problem of physical measurement deals with the means of evaluating certain quantities directly in terms of these units. The instruments and processes mentioned in this chapter being essentially those employed in the experiments which follow, a few words may be devoted to these fundamental measurements.

Length.

Instruments for refined measurements of length usually involve the principle of the micrometer screw or of the vernier, or a combination of the two. Prominent among such instruments may be mentioned the micrometer gauge, the spherometer, the vernier caliper, the cathetometer, the micrometer cathetometer, the comparator, and the dividing engine. Of these the micrometer gauge and the spherometer employ the principle of the micrometer screw, the vernier caliper and the ordinary cathetometer employ the principle of the vernier, while the micrometer cathetome-

ter, the comparator and the dividing engine employ a combination of the two. It is not the intention to describe the working of each of these instruments in detail, but so to give the fundamental principles upon which each instrument is based, as to enable the student to make the application for himself.

The Micrometer Screw.

In the micrometer screw we have a screw of fine thread working in a nut and furnished with a graduated head divided into some convenient number of aliquot parts. A complete rotation of the head advances or withdraws the screw by an amount equal to the distance between its adjacent threads, that is by some fraction of a centimeter, or of an inch. This small length is further subdivided by means of the divisions on the graduated head, so that the "*least count*" of the instrument, that is, the least length directly measurable by it, is the distance between the threads, divided by the number of divisions on the head. Still finer readings may be made by estimating tenths of these divisions.

Exercise 1. The Micrometer Gauge. In the micrometer gauge the end of the screw works against a fixed jaw. The number of complete turns of the screw is read from the stem of the instrument and the fraction of a turn from the graduated head. In use the least count of the instrument is first determined and recorded. The end of the screw is next brought into contact with the fixed jaw by slight pressure and the "zero

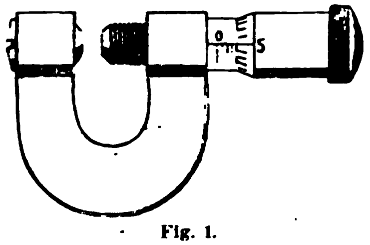

Fig. 1.

reading" taken. The object to be measured is then brought between the jaws and the screw turned down until contact is made as before and the reading is again taken. The difference between this reading and the "zero reading" gives the thickness of the object, *expressed in the units marked upon the stem.* In determining the zero and final readings the mean of five readings is to be taken in each case. In more accurate instruments undue pressure upon the jaws is avoided by means of a ratchet head which slips as soon as contact is made. In using such instruments always turn slowly by means of this head and *stop as soon as the ratchet slips.*

FORM OF RECORD.

Exercise 1.—The Micrometer Gauge. Date..........

Object measured	Pitch of screw ....
Micrometer Gauge No.	Least count ....
Zero readings.	Final readings.
....	....
....	....
....	....
....	....
....	....
Mean	Mean ....

Thickness ....

Exercise 2. The Spherometer.

Fig. 2.

the spherometer the screw works in a nut supported by a frame having three legs of equal length, so placed that when the four points rest upon a plane the three feet form an equilateral triangle about the point of the screw at the center. The instrument is usually placed on a square of good plate glass. Notice of contact between the point of the screw and the plane is given by the instru-

ment's "hobbling" or rocking on the plane. The screw is then carefully turned back until this "hobbling" just ceases. The zero reading is then taken. The object to be measured is then placed beneath the middle point and the screw turned down until contact is made, and the reading taken as before. The difference between the zero and final readings gives the thickness of the object. In the Geneva Society instrument the screw point is connected by a system of light levers, to a delicate pointer which rises when contact is made. Readings are taken when the pointer rises to a fixed mark. In use avoid touching the graduated head with the fingers. Turn by means of the milled head provided for that purpose.

An extremely delicate means of determining the position of contact in the use of the spherometer, is by means of the interference fringes of sodium light. The spherometer is placed upon a piece of good plate glass and a small piece of glass with a good plane surface is placed under the central leg. When the surface of the glass is lighted by a sodium flame, the interference fringes appear at once. The slightest increase in pressure causes the lines to shift their position, thus indicating the position of contact.

FORM OF RECORD.

Exercise 2. The Spherometer. To measure the thickness of a piece of glass.

Spherometer No......... **Date**..........

Pitch of screw........ **Least count**........

Object measured..........

Zero readings **Final readings**

....

....

....

....

....

Mean ____ **Mean** ____

Thickness of..........

The Vernier.

The vernier is a device for subdividing the least division of a scale. It consists of a short subsidiary scale placed in front of, and in contact with the scale of the instrument, and is usually so divided that n divisions of the vernier correspond to n – 1 divisions of the scale. *The "least count" is* 1/n *of a scale division.* Thus, if L be the least division of the scale and V the least division of the vernier, then

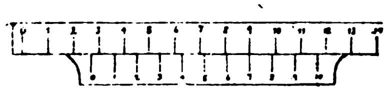

Fig. 3.

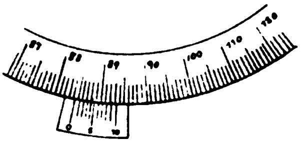

Fig. 4.

$$nV = (n-1)\ L$$

$$V = \frac{n-1}{n}\ L$$

or

$$L - V = \frac{1}{n}L$$

In some cases n divisions on the vernier are made equal to one less than some multiple of n divisions of the scale; the formula then becomes

$$nV = (an - 1)\ L$$

whence

$$aL - V = \frac{1}{n}\ L \text{ as before.}$$

To read the vernier, first read the units of the scale *up to the zero of the vernier:* to this reading add so many *nths* of a scale division as are indicated by the vernier division which coincides with a scale division. Thus in Fig. 3, the reading is 2.7 scale divisions.

Exercise 3. The Vernier.

FORM OF RECORD.

Exercise 3. To determine the least count of the verniers on five different instruments assigned by the instructor.

Date ....

Name of Instrument	Value of L	n	$\frac{1}{n}$ L

Exercise 4. The Vernier Caliper. The vernier caliper (Fig. 5), is an instrument in which the principle of the vernier is applied to the measurement of length. It consists of a pair of steel jaws, one of which is attached to the scale, the other to the vernier which slides upon the scale. In some instruments the movable jaw is provided with a clamp and slow motion screw for fine adjustment. Most instruments are adapted to inside measurements also, by means of a pair of rounded lugs attached to the ends of the jaws. When a separate scale is not provided for inside measurements, the thickness of these lugs must be added to the indicated reading.

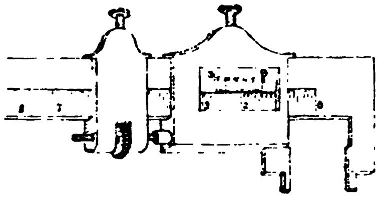

Fig. 5.

In use the value of a scale division L, and the least count, are first determined and recorded. The jaws are next brought together and the zero reading taken. The object to be measured is then placed between the jaws and the mean of several readings taken. The difference between the zero and final readings gives the length of the object.

FORM OF RECORD.

Exercise 4. The Vernier Caliper. To measure the diameter and length of a brass cylinder.

Object measured .... Date ....
Vernier Caliper No. .... Least count ....

Zero readings	Diameter readings	Length readings
....	....	....
....	....	....
....	....	....
....	....	....
Mean	Mean	Mean ....

Diameter .... Length ....

Line Measurements.

In the exercises on measurements of length it will be noted that in each case the measurement has been effected by *end measurements*, that is, by bringing the measuring instrument against the ends of the object measured. Frequently this is neither practicable nor desirable. In such cases *line measurements* are employed; that is, the distance between two lines, drawn upon a body is determined, by focussing a microscope or telescope upon the lines in succession, and noting the readings of the vernier attached to the instrument. Line measurements are made by means of the cathetometer, comparator and the dividing engine, and are used in all cases where great accuracy is desired.

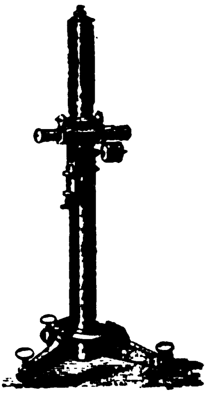

Fig. 6.

The Cathetometer.

The cathetometer is an instrument for measuring the difference in height between two points. It consists of a vertical standard, (Fig. 6), upon which slides a ring carrying a telescope and furnished with a vernier and a clamp for holding the

telescope at any height desired. The instrument is provided with a level for bringing the axis of the telescope into a horizontal line, by means of adjusting screws. In most instruments the level may be rotated through a right angle about the standard as an axis, in order to secure the vertical position of the latter. The telescope is focussed first upon one point and the reading taken, and then lowered by means of the clamp to the level of the second point, and set upon it, and the corresponding reading determined. For ease and accuracy of making the settings, the ring carrying the telescope is usually furnished with a slow motion screw. The difference between the two settings gives the vertical distance between the two points.

In the micrometer cathetometer the telescope is replaced by a microscope of low power, and the vernier is combined with a micrometer screw. By this means the distance between two very near points may be determined with great accuracy. This instrument is employed in the experiment for determining Young's modulus by stretching.

Mass.

Strictly speaking all determinations of mass are indirect. The *balance* is an instrument for the comparison of masses. In its simplest form it consists of a light beam turning readily about its middle point and carrying at its ends two scale-pans of equal weight. When disturbed the system oscillates about its position of equilibrium to which it finally returns. When masses are placed in the pans, it is evident that the original position of equilibrium will be resumed, only when the moments of the forces due to the action of gravity upon these masses are equal. If now, we assume the arms of the balance to be equal,

we may set the masses equal to each other when their moments have been shown to be equal; i. e., when the balance resumes its original position of equilibrium.

In balances of precision (Fig. 7), the beam and scale pans are hung from accurately ground knife-edges resting upon agate plates. When not in use, the knife-edges are relieved from the weight of the pans and beam by means of an arresting device *This must always be used when weights are to be changed, or articles to be weighed are to be placed upon the scale pans or removed from them.*

Fig. 7.

The oscillations of the balance are observed by means of a long slender pointer moving in front of a graduated scale. Care should be taken to raise the system from the knife-edges, only when the pointer is at the middle of the scale. A system of light levers is usually placed under the pans, to maintain the balance in equilibrium for small differences of weight in the pans, and to prevent undue movements of the beam during rough weighing. The beam is generally divided from the middle outward, into n equal divisions, the last one coinciding with the knife-

edge over the pan. By sliding upon this beam a small wire rider weighing n milligrams, weighings may be made directly to milligrams. For subdivision of the milligram the method of oscillations is used.

Determination of the Resting Point.

Owing to the loss of time incident upon waiting for the balance to come to rest, it is usual to determine the final position of the pointer from a series of its successive turning points. Since the vibrations of the system are more or less damped, it is necessary to take the initial and final readings of the pointer on the same side. If we call the central division of the scale zero, and readings to the right and left respectively plus and minus, then the resting point is found by averaging the means found for each side separately. For example, if the readings are

	Left		Right
	−9.2		+10.4
	−8.9		+10.0
			+9.7
Mean	−9.05	Mean	+10.03

$$\text{Resting point} = \frac{-9.05 + 10.03}{2} = +0.49$$

This means that the pointer would finally come to rest at a point about 0.5 of a division to the right of the zero. The resting point should be determined both before and after making a weighing, and should remain constant if the balance be properly adjusted.

In some balances the scale divisions are numbered continuously from left to right. In the use of such instruments the readings are taken directly and the positive and negative signs are avoided.

Sensibility.

If the rider be now placed upon the first division of the beam and the resting point determined as before, then the difference between these two resting points is the number of scale divisions through which the beam has turned for a difference in weight of one milligram. This is by definition the sensibility of the balance, and is usually termed the *sensibility for zero load in the pans.* Since the sensibility varies with the load, it is always necessary to determine it for the specific load upon the pans.

To Make a Weighing.

First determine the resting point for zero load. Next place the object to be weighed upon the left hand scale pan and an estimated equivalent weight upon the right hand pan. Release the arrest very slightly and note the indication of the pointer. If the weight be too small it should be sufficiently increased to turn the pointer to the opposite side on the next trial. In this way the true weight may be rapidly approximated to the nearest gram, then to the nearest centigram. After this the *balance case should be closed,* the rider applied and the pan arrests turned down. Having found the weight to the nearest milligram, the balance is set swinging and the resting point is determined. The rider is then shifted one whole division on the beam, so as to bring the resting point on the other side of the zero position, and the resting point again determined. If now we call the three resting points p . p, and P, where P corresponds to the weight *greater* than the true weight, then dW, the fraction of a milligram to be added to smaller weight is the $\frac{p - p}{p - P}$. Thus suppose

$p_0 = +0.49$, $p = +0.77$, $P = -0.15$, then $\frac{p - p_0}{p - P} = \frac{0.28}{0.92}$
$= 0.3$ milligram.

Care should be taken to avoid error in counting up the weights upon the scale pan. An excellent method is to write down the weights from the vacant spaces in the box and check each weight as it is returned to the box.

Reduction to Weight in Vacuum.

The weight of a body in vacuum is

$$W = w\left(1 + \frac{a}{d} - \frac{a}{D}\right)^{*},$$

where w is the observed weight of the body, d its density, and a and D the densities of the air and the weights respectively. In weighing a quantity of water with brass weights, this correction amounts to more than 1 milligram, for every gram.

Exercise 5. Double Weighing.† In order to eliminate any inequality between the two arms of the balance the object may be weighed first in the left hand pan and then in the right. The true weight, W_0, is given by the equation

$$W_0 = \sqrt{W_1 W_2}.$$

This precaution is necessary only in very accurate work. Would double weighing be of any advantage in operations involving relative weights?

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* For derivation of this formula see Carhart's University Physics, **Part I, page 132.**

†**For more** detailed treatment of the balance and its use see **Stewart & Gee, Vol. I, pp.** 63-94; also Carhart, I. pp. 75 78.

FORM OF RECORD.

Exercise 5. Weigh a piece of brass, applying the method of double weighing.

Balance No....................... Date ....

Object to be weighed ....

Zero load.		Load left.		Load right.	
L.	R.	L.	R.	L.	R.
....	....	....	....	....	....
.....	....	....	....	....	....
	....	....	....		....
p_0		p		p'	

* * * *

Rider removed one point to the right in each case.

$W_1 + 1$ mg.		$W_2 + 1$ mg.	
Load left.		Load right.	
$W + 1$ mg.		$W_2 - 1$ mg.	
L.	R.	L.	R.
...	..	..	....
....	..	....	......
P		P'	

Let dW_1 and dW_2 be the fraction of a mg. to be added to W_1 and W_2 respectively,

then $$dW_1 = \frac{p - p_0}{p - P} = \ldots, \quad dW_2 = -\frac{p' - p_0}{p' - P'} = \ldots$$

It is to be observed that in the case of dW_2, the fraction $\frac{p' - p_0}{p' - P'}$ *is negative*, since $P' > p'$.

Finally $$W_0 = \sqrt{W_1 W_2}$$

Time.

Most measurements of time in the physical laboratory consist in the determination of the period of some vibrating body, as a pendulum, magnetic needle, galvanometer needle, etc. The most common problem is that of rating a pendulum, and what follows is applied directly to this end, although the method is equally applicable to the case of any freely vibrating body.

The process usually adopted is a modification of Borda's method of coincidences. The essence of the method consists in determining the time necessary for the pendulum to complete some large number of vibrations; from this, if the number of vibrations be known, the period of a single vibration is at once deduced. Perhaps the simplest way is to note the time occupied in counting 100 or 1000 vibrations. This method however, is both tedious and inaccurate, since owing to its monotony the observer is liable to make an error of one or even of ten vibrations in counting a large number. To this source of error is added the uncertainty of beginning and ending the count exactly upon the second.

In order to avoid the first source of error the pendulum is made to keep count of its own vibrations when compared with a clock beating seconds. The second error is minimized by attaching a pointer to the pendulum and observing its passage over a scale, or in work of greater accuracy, by viewing the pendulum through a telescope and noting its passage over the cross-wires in the focal plane of the eye-piece. It is best to observe the passage when the pendulum is in the middle of its swing, and moving with its maximum velocity. The clock is connected electrically with a sounder and the beats are thus made audible throughout the room.

The pendulum to be rated having been set swinging through a small arc, the observer at the telescope notes the transits of the pendulum image from left to right over the cross-wires and awaits the coincidence of such a transit with the click of the clock. He then notes carefully the number of seconds elapsing before the next passage of the pendulum over the cross-wires *in the same*

direction and estimates, as well as possible, the fraction of a second in tenths, thus gaining a roughly approximate period. Suppose the period is found to be somewhere between 2.3 and 2.5 seconds.

A coincidence is again observed and the seconds counted continuously until a number of fairly good coincidences have been observed. From this observation a closer approximation to the period can be obtained. Thus if the period is near 2.5 seconds, good coincidences will occur in 5, 10, and 15 seconds, i. e. after 2, 4 and 6 complete vibrations of the pendulum. If on the other hand the period be nearly 2.3 seconds, then fairly good coincidences will be noted at 7 and 16 seconds, and a good one at 23 seconds, the intervals corresponding to 3, 7, and 10 vibrations. The imperfect coincidences at 7 and 16 seconds are due of course to the fact that the interval of 7 seconds is 0.1 second greater than the time needed for 3 swings, and that of 16 seconds is less by 0.1 second than that needed for 7 swings of the pendulum. In this way it is possible to determine the provisional period accurately to tenths of a second.

Several trials should be made and 50 or even 75 seconds counted, if necessary to determine this with accuracy. Suppose it has been found to be 2.3 seconds. The observer again awaits a good coincidence, noting the seconds by calling each one *up to and including the second of coincidence*, "zero." Then he walks to the clock, counting the seconds as he goes, and on the tenth second reads the time, noting the seconds first, then the minutes and then the hour. This recorded coincidence is obviously ten seconds later than the observed one, but by counting *ten seconds* each time before reading the

clock, the interval between the coincidences is preserved and no error is introduced.

A second coincidence is observed as soon as possible and recorded in the same way. Now the difference between the first and second readings of the clock gives the number of seconds corresponding to some *integral* number of swings of the pendulum, and a glance at the approximate period is usually sufficient to show what this number is. In case of doubt divide the time by the provisional period and take the nearest integer as the number of vibrations. (Why?) The number of seconds divided by the number of vibrations, gives the period to a closer degree of approximation than before. A third coincidence is observed and recorded as before. The interval *between this coincidence and the first* corresponds to a still larger number of integral swings of the pendulum. This larger number is found by dividing the seconds by the *period last deduced*, and the new value of the period is computed as before. Thus by successive observations we find intervals corresponding to a larger and larger number of vibrations, *using in each case the period last found.*

In this way the period of a pendulum may be readily and rapidly determined to thousandths of a second. After a little practice the student is able to judge a coincidence accurately to 0.1 of a second. In such a case after 20 minutes of observation our pendulum would have made something over 500 vibrations, and the time needed for this number of vibrations would be determined to ± 0.2 of a second, or the period would be accurate to thousandths of a second. The following example will make the method and computation clear:

SIMPLE PENDULUM.

February 8th, 1899.

Length, 130 cms.

Good coincidences 7, 14, 16, 23.

Approximate period 2.3 seconds.

Transits.		Seconds.	Vibrations.	T.
3 h. 19 min.	52 sec.			
20	15	23	10	2.3
20	38	46	20	2.300
21	26	94	41	2.293
(21	50)	118	52	2.269 ??
22	30	158	69	2.289^{8}
23	57	245	107	2.289^{7}
24	29	277	121	2.289^{2}
30	19	627	274	2.288^{3}
30	51	659	288	2.288^{1}
33	22	810	354	2.288^{1}

It is to be observed that the period T gradually approaches a limiting value which becomes constant to thousandths of a second as soon as the number of observed vibrations, n, reaches a definite value. If the maximum error, e, made in taking any coincidence be ± 0.1 of a second, then the maximum error possible in any number of observed seconds is 0.2 of a second. Hence to have the period T constant to thousandths of a second, we must make $\frac{0.2}{n}$ less than 0.0005, or n must be greater than 400. Obviously n must be larger, the larger e becomes. How large must n be taken if $e = \pm 0.2$ seconds?

In case any observation gives a result sharply at variance with the others, the difficulty lies either in the arithmetical work, or in a false reading of the clock. The latter error renders the observation useless; it should be bracketed as indicated above, and the next taken with greater care. The advantage of the method is that no

single error in reading the clock can permanently vitiate the result.

Instead of determining the number of vibrations by dividing the seconds by the last value of T deduced, it is much simpler to use the preceding intervals and vibrations as measures of the new. Thus the second interval 46, is manifestly double the first; hence the number of vibrations must be twice as many, or 20. In the third, the interval, 94 seconds, is twice the second +2 seconds; the excess, 2 seconds, corresponds to an additional vibration; hence $n = 2 \times 20 + 1 = 41$. In the seventh determination the interval 277 seconds, may be evaluated for n in a number of ways; thus from the third we may have

$$\begin{array}{rcr} \text{secs.} & & \text{vibs.} \\ 282 & = & 123 \\ 277 & & \\ \hline -5 & = & -2 \\ \hline 277 & = & 121 \end{array}$$

or from the third and fifth thus:

$$\begin{array}{rcr} 94 & = & 41 \\ 158 & = & 69 \\ \hline 252 & = & 110 \\ 277 & & \\ \hline +25 & = & +11 \\ \hline 277 & = & 121 \end{array}$$

Exercise 6. The Pendulum.

FORM OF RECORD.

Exercise 6. Determine the period of a torsional pendulum, to 0.001 of a second.

Record as indicated above.

For the measurement of small intervals of time the tuning fork furnishes an accurate and convenient method. For the practical application of this method see subsequent articles.

Exercise 7. The Barometer. The barometer, (Fig. 8), consists of a closed tube of glass of uniform bore about 80 cms. long, filled with mercury and inverted in a dish containing mercury. The free surface of the mercury in the vessel is in communication with the outer air. In the cistern barometer, the reservoir (Fig. 9), has a bottom of leather which is adjustable by means of a thumb screw. A small ivory point extending downward from from the upper surface of the reservoir, forms the zero-point from which the measurements indicated on the scale are reckoned. Before reading the barometer the mercury in the cistern must be so adjusted by means of the screw that the surface just touches the ivory point. The upper part of the tube is then gently tapped to free the mercury surface from the sides of the tube, and the vernier adjusted by means of the thumbscrew at the side, until, on looking through the slit in the barometer case, the upper part of the meniscus is seen to be just tangent to the line joining the sharp edges at the front and back of the vernier.

Fig. 9.

Fig. 8.

In the instrument made by Haak, of Jena, we have an automatic adjustment of the mercury in the cistern. The zero point is the tip of a vertical tube connecting with a lower, auxiliary reservoir. Air is forced into the lower

reservoir by means of a bulb, thus driving mercury into the reservoir proper and covering the zero point. On releasing the bulb the mercury flows out until the tip of the zero point tube is again exposed; the barometer is then in adjustment for reading. The scale is etched directly upon the tube of the barometer, and fractions of a millimeter may be read with ease by means of the adjustable ring, at the top, which carries a fine line for subdividing the millimeter divisions.

Readings on the barometer must be corrected for temperature effects, and are reduced to 0° C., by the use of the following formula:

$$H_0 = H_t\ [\,1 - (\,a - b\,)\,t\,], \quad * \qquad (1)$$

where H_0 is the barometric height at 0° C.; H_t is the barometric height at t° C.; a is the cubical coefficient of expansion for mercury, (a = 0.000181), b is the linear coefficient of expansion for the material of the scale; (b for glass = 0.0000085, for brass, b = 0.000019).

FORM OF RECORD.

Exercise 7. Adjust and read each barometer three times: correct for temperature and compare readings.

Barometer No. 1.	Barometer No. 2.	Date ....
Least count	Least count	
Readings.	Readings.	
....	....	
....	....	
....	....	
Mean	Mean	
Reduced to 0° C ...	Reduced to 0° C	

Express the barometric pressure in dynes per square centimeter.

---

* For reducing the barometric reading to standard conditions, viz: 0° C, sea level, in latitude 45° we have the complete formula

$$H = H_t\ \frac{g}{g_{45}} \cdot \frac{1 + bt}{1 + at}\,.$$

CHAPTER II.

DENSITY.

Exercise 8. Density from Mass and Volume.

FORM OF RECORD.

Exercise 8. To determine the density of a brass cylinder from its volume and mass.

Density of brass cylinder No .... Date ....

Length.	Diameter.	
....	....	
....	....	Mass ....
....	....	
....	....	
....	....	
Mean	Mean	
	Volume	Density ....

Exercise 9. The Pyknometer. The pyknometer in its simplest form consists of a glass vessel (Fig. 10), provided with an accurately ground stopper, perforated throughout its length. Before use it is to be thoroughly cleaned and dried with alcohol or ether. It is then carefully weighed. Call this weight W_0. The pyknometer is then filled with distilled water, placed in a bath of water at 30°C. and allowed to remain there five minutes. (Why?) It is then wiped dry, and its weight, W_1, determined. After cleaning and drying as before, the pyknometer is filled with the liquid

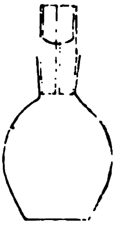

Fig. 10.

under examination, brought to the temperature of 30°, wiped dry and weighed. Call this weight W_2. Derive and apply formula for the density of a liquid. Correct for temperature, by multiplying s, the value found, by D, the density of the standard.* (at 30°C., D = 0.9957). Unless the values are to be carried to more than three decimal places, the buoyant force of the air may be neglected. Instead of making all the weighings at 30°, any other temperature may be taken provided it be higher than the temperature of the room where weighings are made. (Why?)

FORM OF RECORD.

Exercise 9. Determine the density of two liquids.

Pyknometer No. . . . . Date . . . .

Density of . . . .

W_0 = ...
W_1 = ... s = ...
W_2 = ... D = ...
Corrected density, d, of . . . . = ...

Exercise 10. Mohr's Balance. In Mohr's balance, (Fig. 11), the beam is divided into ten equal parts of which the last coincides with the end. Upon this end is hung by means of a fine platinum wire a small sinker containing a thermometer. The weights consist of four pairs of riders weighing respectively m, 0.1 m, 0.01 m, 0.001 m grams. The instrument is usually so adjusted that the sinker is exactly counterpoised in air. When immersed in water at 4°C. the buoyant force on the sinker

Fig. 11.

*See Carhart's University Physics, Vol. I, p. 111.
Also Stewart and Gee, Practical Physics, Vol. I, p. 119.

is equal to the weight of the largest rider. When the sinker is put into any other liquids the weights needed to equal the buoyant force upon the sinker in each case are in direct proportion to the densities of these liquids. If m be called unity then the density of the liquid can be read directly from the beam. The balance must show for water at t°C. the density corresponding to this temperature as given in table II. If, instead of this density d, it show d_1, then all results must be multiplied by $\frac{d}{d_1}$. It is obvious that the instrument may be used as an ordinary balance as well, and the densities of solids and liquids referred to water at any temperature may be determined by means of it with equal ease.

FORM OF RECORD.

Exercise 10. Redetermine the densities of the substances used in exercises 8 and 9.

1. For solids: Date ....
   Density of ....
   Weight in air ....
   Weight in water .... Temperature of water ....
   s. ....
   Corrected density ....

2. For liquids .... Date ....
   Density of .... Density ....
   Temperature ....
   Buoyancy of sinker in water ....
   Buoyancy of sinker in liquid ....

CHAPTER III.

ELASTICITY.

Elasticity is that property by virtue of which matter resists the action of a force tending to change its shape or bulk, and which causes it to resume its original shape or bulk after the force is removed. If a body possess elasticity of shape it is called a solid; if it possess no elasticity of shape it is called a fluid.

Fluids possess perfect elasticity of bulk, i. e., they return exactly to their former bulk on removal of the compressing force. Solids do not all recover their initial shape with equal promptness. In some cases the return is much retarded, especially after repeated or long continued distortion. This is commonly termed *elastic fatigue*, and is quite noticeable in metals. For every solid there is a limiting distortion beyond which the body, when freed from the distorting force, no longer completely regains its former shape. This is called its *limit of elasticity*.

HOOKE'S LAW.

Any change in a solid either of size or shape produced by the action of a force is called *a strain*. The force producing such a change is called *a stress* and is measured in

dynes per unit area of the cross-section upon which the stress is exerted. When an elastic body is distorted within its limit of elasticity, the opposing force called out by the distortion, tending to restore the body to its original condition, is proportional to the distortion. This is known as "Hooke's Law," and as originally stated, "*ut tensio sic vis,*" expresses the proportionality between the distortion and the restoring force. The applications of this law are very numerous including every form of elastic reaction against strains produced by external mechanical agencies.

Coefficients of Elasticity.

In general twenty-one coefficients would be needed to express completely the elastic nature of any solid. If however, the solid be isotropic, these twenty-one coefficients reduce to two: *the coefficient of volume elasticity, e, and the coefficient of simple rigidity, n*. The general expression for these coefficients is the quotient arising from dividing the *stress by the strain.*

Coefficient of Volume Elasticity.

In the case of the coefficient of volume elasticity e, we have the stress or applied pressure p, divided by the compression produced, where compression denotes the change in volume v, divided by the original volume V, or

$$e = \frac{p}{\frac{v}{V}} = \frac{p}{v}V. \qquad (2.)$$

In the case of a gas, the volume is at all times a function of the pressure to which it is subjected. Hence for gases it should be noted, that the coefficient of elasticity is to be defined in terms of the *change in pressure* and the corresponding *change in volume.*

Since these changes are conceived as being very small if we assume a volume of gas V, to be subjected to a change in pressure dp, producing a corresponding change in volume dV, then for a gas,

$$e = V\frac{dp}{dV}. \tag{3}$$

It should be observed that the expression for the strain, $\frac{dV}{V}$, denotes a *dilatation if positive* and a *compression if negative.* The coefficient e, however, has reference simply to the absolute value of the ratio $V\frac{dp}{dV}$, and is therefore independent of the sign. The coefficient of elasticity of volume is the only one possessed by fluids, and is of special interest in all cases involving the propagation of disturbances through fluid media.

YOUNG'S MODULUS.

In solids in the form of wires or rods, subjected to longitudinal stresses tending to produce either elongations or compressions, we are interested in the relative elongation or compression l, produced in length L, and cross-section a, by a force of F dynes, when the body is free to contract or expand laterally. In general, longitudinal expansion is accompanied by lateral contraction and longitudinal compression by lateral distention. The *measure* or *modulus* of the elastic behavior of a solid under such conditions is known as Young's modulus, and may be defined as the ratio between longitudinal stress per unit area $\frac{F}{a}$, and longitudinal strain per unit length $\frac{l}{L}$; that is

$$M = \frac{FL}{al} \tag{4}$$

or more specifically, *Young's modulus is that force which, when applied to a wire of unit cross-section, would be sufficient to stretch it to double its length*, provided of course, that the elongation remain proportional to the force at all times.

Coefficient of Simple Rigidtiy.

Besides the elasticity of volume, solids have, as we have seen, elasticity of shape as well. If a solid be so distorted that its shape alone is changed, it is said to have undergone *a shear*. Thus if we conceive all the particles in one plane in a body to be fixed, and all the remaining particles to move in planes parallel to this plane, and by amounts proportional to their distances from this plane, *such a motion constitutes a shear*. The stress applied to cause a shear is called *a shearing stress*, and the coefficient of simple rigidity is the quotient obtained by dividing the shearing stress per unit area by the shearing strain per unit length.

In order to learn how these quantities may be experimentally determined, let us consider a circular cylinder (Fig. 12), of radius r, held vertically by a rigid clamp at the upper end and subjected to a torsional twist at the lower end. The effect of such a twist is to produce a shearing strain throughout the cylinder. Imagine the cylinder to be made up of a large number of tubes, one inside the other, and cut at right angles to the axis into a large number of circular sections. Each circular section would be composed of a large

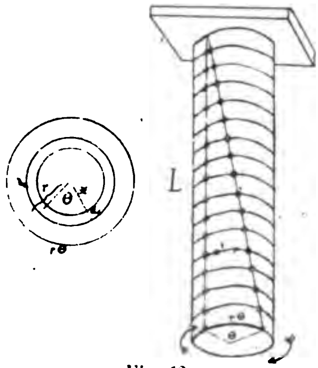

Fig. 12.

number of concentric rings. Now the shearing strain increases regularly from above downward from section to section, and when the lower end of the cylinder has been twisted through an angle θ, the shear for the *outer ring of the lower section will be the arc $r\,\theta$, and the shear per unit length* will be $\frac{r\theta}{L}$; or in general, the shearing strain per unit length at any point in a cylinder is the circular displacement at that point divided by the distance from that point to the fixed end of the cylinder.

Again since the cylinder under stress is in equilibrium, the moment of the couple producing the shear must be balanced by the moment of the couple called out by the shear; or by Hooke's law, it must be proportional to the shear itself, hence also proportional to $\frac{r\theta}{L}$. Now by definition, the coefficient of simple rigidity n, is the proportionality factor between shearing stress and shearing strain; hence we have the *shearing stress per unit area* $= \frac{nr\theta}{L}$.

Let us now consider the entire lower circular section of the cylinder, and in that section, a ring of radius x and of width dx. The shear per unit length will be $\frac{x\theta}{L}$, and the shearing stress per unit area will be $\frac{nx\theta}{L}$. The area of the elementary ring is $2\pi x\,dx$, hence the total shearing stress for the ring is $\frac{2\pi n\theta x^2 dx}{L}$. This force acting with a lever arm of x, gives an elementary moment of shearing stress for the elementary ring of width dx, equal to

$$d\mathcal{T} = \frac{2\pi n\,x^3\,dx}{L}. \qquad (5)$$

For the entire section, the moment of the shearing stress will be the sum of the elementary moments for all elementary rings, whose radii vary from 0, at the center, to r at the surface of the cylinder, and so the moment of the torsional couple called out by the shear, is found by integrating the expression in equation (5), or

$$\mathcal{T} = \int_0^r \frac{2\,\pi\,n\,\theta\,x^3\,dx}{L} = \frac{2\,\pi\,n\,\theta\,r^4}{4L}$$

or

$$\mathcal{T} = \frac{\pi\,n\,\theta\,r^4}{2L} \qquad (6)$$

whence

$$n = \frac{2\,L\,\mathcal{T}}{\pi\,\theta\,r^4}. \qquad (7)$$

It must be observed that θ is here expressed in *radians*. If θ is measured in degrees, what correcting factor must be introduced? In the expression

$$\mathcal{T} = \frac{\pi\,n\,\theta\,r^4}{2L}$$

$\mathcal{T}$ is defined as the moment of the torsional couple producing an angular twist θ, in a cylinder of radius r, and length L. By making the angular twist equal to unity we have

$$\mathcal{T}_1 = \frac{\pi\,n\,r^4}{2L} \qquad (8)$$

and finally, by reducing the length L and the angular twist θ, each to unity we have

$$t = \frac{\pi\,n\,r^4}{2} \qquad (9)$$

The quantity t is called *the modulus of torsion: it is the couple required to produce unit angular twist in a wire of unit length.*

Summary :—We have now defined and derived expressions for the following quantities :

$$e = V\frac{dp}{dV} \qquad (3)$$

$$M = \frac{FL}{al} \qquad (4)$$

$$\mathcal{T} = \frac{\pi n \theta r^4}{2L} \qquad (6)$$

$$n = \frac{2L\mathcal{T}}{\pi \theta r^4} \qquad (7)$$

$$\mathcal{T}_1 = \frac{\pi n r^4}{2L} \qquad (8)$$

$$t = \frac{\pi n r^4}{2} \qquad (9)$$

In the experiments which follow several of the above quantities will be measured in one or more different ways.

Exercise 11. To Verify Boyle's Law. The apparatus consists of a cylindrical reservoir, (Fig. 13), formed of a glass tube some 25 cms. long and 3 cms. in diameter, into which is sealed a uniform tube B, some 30 cms. long and 12 mm. in diameter, closed at the top by a square-cut plug carefully cemented in, and at its lower end extending to the bottom of the larger tube. At the lower end of the reservoir is sealed on a second tube A, 3 mms. in diameter and about 200 cm. long. This tube is bent back upon itself about 10 cms. below the reservoir, so as to be vertical and parallel to the tube B. It is terminated at its upper end by a small thistle bulb, for convenience in filling the

Fig. 13.

reservoir with mercury. The instrument is mounted upon

a suitable support carrying a scale graduated to millimeters at the side of the tube A, throughout its entire length, and at the bottom extending between the two tubes so that readings upon the height of the mercury in the tubes A and B, may be made from opposite edges of the same scale. A small side tube is sealed to the reservoir near the top by means of which air may be forced into the reservoir from a small force pump. Before beginning the experiment the instrument is so adjusted that the shorter tube B, is about half filled with mercury when the air in the reservoir is at atmospheric pressure. Air is next driven in through the side tube C, by means of the pump, until the mercury almost fills the long tube A. The air in B is now under a pressure measured by the barometer column plus the difference in height of the mercury in the tubes A and B, and is correspondingly compressed.

Varying pressures and corresponding volumes are successively secured by allowing small quantities of air to escape through the tube C. Readings are made upon the height of the mercury in A and B, and upon the lower end of the plug in B. Readings should be continued until the mercury in the tube A falls to the level of the mercury in the reservoir. The air must be allowed to come to the temperature of the room after each setting before the reading is taken. (Why?)

According to Boyle's law, the product of the pressure and the volume of a gas is a constant, *for a constant temperature*, * or

$$p \, . \, V = C \, . \qquad (10)$$

* See Carhart's University Physics, Vol. I. p. 128.

By definition the coefficient of volume elasticity for a gas is

$$e = -V\frac{dp}{dV}.$$

Also from (10) by differentiation we get

$$pdV + Vdp = 0, \quad \text{or} \qquad (11)$$

$$p = -V\frac{dp}{dV} = e. \qquad (12)$$

Thus we see that for a perfect gas undergoing isothermal changes, the coefficient of volume elasticity e, is at all times equal to the pressure p. For the purpose of our experiment let a and b denote the observed heights of the mercury in the tubes A and B. Let P represent the atmospheric pressure at the time of the experiment, $p = a - b$, the applied pressure, and let V represent a quantity proportional to the resulting volume of the air enclosed in the tube B. Then equation (10) becomes

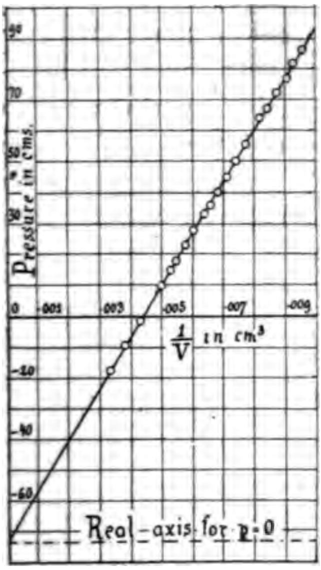

Fig. 14.

$$V(P + p) = C, \qquad (13)$$

or

$$P + p = C/V. \qquad (14)$$

If now we plot the observed values of p on the Y axis and the reciprocals of the corresponding volumes on the X axis, (see Fig. 14), our curve is represented by the equation

$$y = cx - P, \qquad (15)$$

the equation of a straight line cutting the Y axis at a point

P below the origin. This point may be considered the true origin, measured from which the values of y denote the successive values of e, corresponding to the related values of V. The tangent of the angle included between the curve and the X axis is equal to C. If all quantities be expressed in the proper units, then C becomes *the gas constant*. (Under what conditions does e approach zero?)

FORM OF RECORD.

Exercise 11. To Verify Boyle's Law.

Reading on plug...... Barometer...... Date........

Tube A	Tube B	p	a — b	V	colog V	1/V

Plot values of p and 1/V. Determine the value of P from the curve and compare the result with the barometer reading at the time of the experiment.

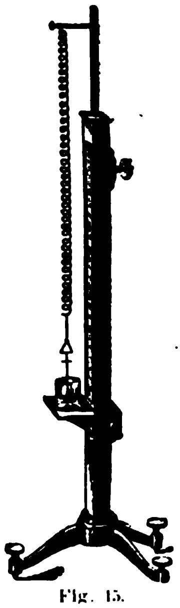

Fig. 15.

Exercise 12. Jolly's Balance. In the Jolly's balance (Fig. 15), the elastic body is a spiral spring mounted in front of a mirror scale and carrying on its lower end a pan to receive the substances under experiment. To this is attached a second pan to carry the substance when in the water. Readings are made from some convenient fixed point on the spring or pan, usually a sharp hook or a bright bead, whose image in the mirror forms a distinct object for the eye. Parallax is avoided by bringing the object and its image into coincidence before reading the scale. The reading is greatly facilitated by bringing a white card pierced with a small hole, 3 mms. in diameter, close up before the eye, and

standing in front of the scale at a distance of about two feet, so that the object and its image are both seen sharply focussed by the eye at the same time. An adjustable support carries a small vessel containing the distilled water.

Before *every* reading the support is so adjusted as to bring the lower pan to a certain depth in the distilled water. This is most readily done by having a small semicircular hook on the upper pan which is brought into contact with the upper surface of the liquid. On looking obliquely upward from below the hook and its image may be brought into contact so as to form a complete circle. In this way the setting may be made with great accuracy.

In use the support is adjusted and the reading r_c, taken with both pans empty. The substance under experiment is then placed in the upper pan, the support adjusted and a second reading r_1, is made.

The substance is then transferred to the lower pan, the adjustment made and the reading r_2, taken. Then for the density of the substance we have

$$D = \frac{r_1 - r_c}{r_1 - r_2}. \qquad (16)$$

Derive and explain this formula

For substances lighter than water a small piece of metal heavy enough to sink the substance, is placed in the lower pan and kept there during all three readings. The density is then computed as above.

For liquids the lower pan is removed and a suitable sinker, usually of glass is attached to the hook by a fine platinum wire. Readings are made with the sinker in air, r_c, with the sinker immersed to a definite depth in the water, r_1, and with the sinker immersed to the same

depth in the liquid, r_2. Derive and explain the formula in this case.

$$D = \qquad (17)$$

It is necessary to exercise care in the use of the apparatus in order to secure trustworthy results. Air bubbles must be removed from the substance and from the lower pan before readings are taken. Do not allow the upper pan to come in contact with the water at any time. Determine the density of three solids: brass, zinc and paraffin, and of a solution of copper sulphate. Compare results with those obtained in preceding exercises.

FORM OF RECORD.

Exercise 12. Density by means of Jolly's balance.

Density of .... Date ....

	r_0	r_1	r_2	
	....	....	....	
	....	....	....	Density ....
Mean	....	....	....	

Exercise 13. Young's Modulus by Stretching. An iron bracket firmly attached to the wall near the ceiling supports a long brass wire which carries on its lower end by means of a clamp, a cage for the reception of weights. Near the lower end of the wire is attached a needle upon the point of which is focussed a micrometer cathetometer reading to 0.001 mm. In order to eliminate the yielding of the supporting bracket, a small rod hung from it is loosely attached to the wire, and bears on its lower end a small metal square or flag, near the needle point, so that both flag and point appear in the field at the same time and readings may be made upon them successively. An adjustable table is placed under the cage before weights

are added and then gradually lowered, to avoid subjecting the wire to sudden jerks. A weight of two kilograms is left permanently upon the cage in order to free the wire from kinks and to insure uniformity of stretching.

Readings are taken upon the flag and the point, F_o, P_o, with only the zero load, two kilograms, on the cage, the mean of three readings being used in each case.

Let $$F_o - P_o = l_o\,.$$

The table is then raised beneath the cage and two kilograms added. The table is then lowered and readings are made as before.

$$F_1 - P_1 = l_1\,.$$

In this way readings are successively taken for weights of four and six kilograms in addition to the zero load. After this the weights are removed, two kilos at a time; readings being made as before until the zero load remains. From the values thus found the stretch for two kilograms is computed for each reading. Thus from the reading with 6 kilograms added we have

$$\frac{l_3 - l_o}{3} = l\,.$$

The mean of the values of l thus obtained, is taken as the stretch produced by two kilos. Having determined L and a, we have

$$M = \frac{F\,L}{a\,l} = \frac{2000\ .\ 980\ .\ L}{a\ l} \text{ dynes per cm}^2.$$

FORM OF RECORD.

Exercise 13. Young's modulus by stretching.

L............ Date............

r............ $a = \pi r^2$................

Weights	Flag.	Point.	l_{11}	l	Computation:
0					log 2000 = ...
2					log 980 = ...
4					log L = ...
6					colog πr^2 = ...
4					colog l = ...
2					log M = ...
0					
			Mean....		M = ...

Exercise 14. Verification of the Laws of Bending. The bending of beams supporting a weight is, as is well known, a function of the weight supported, W, of the three dimensions of the beam, l, b, d, and proportional to a constant C, which depends upon the material of the beam and the manner in which it is supported. It is proposed in this exercise to determine experimentally these relations. Let us assume for the purposes of the investigation, the general expression for the bending B,

$$B = C\,w^{\alpha}\,l^{\beta}\,b^{\gamma}\,d^{\varepsilon}, \tag{18}$$

where α, β, γ, ε, are constant exponents to be determined by experiment. Since this expression is perfectly general it will include all possible bendings of the bar, obtained by varying the weight, length, breadth, or depth of the bar, either successively or simultaneously. In order to keep clearly in mind the relations under investigation we shall vary but one of these quantities at a time, and observe the bendings produced under definite conditions. *Thus* [illegible] of definite length, breadth and depth,

we shall *vary the load* successively and observe the bendings B_1, B_2, B_3, B_4, B_5, produced by the loads w_1, w_2, w_3, w_4, w_5.

Inserting related pairs of values of B and w, as B_1 and w_1, B_2 and w_2 in our general formula we have

$$B_1 = C\, w^{\alpha}_1\, l^{\beta}\, b^{\gamma}\, d^{\varepsilon}$$

$$B_2 = C\, w^{\alpha}_2\, l^{\beta}\, b^{\gamma}\, d^{\varepsilon}, \quad \text{and so forth.}$$

Passing to logarithms we have

$$\log B_1 = \log C + \alpha \log w_1 + \beta \log l + \gamma \log b + \varepsilon \log d$$

$$\log B_2 = \log C + \alpha \log w_2 + \beta \log l + \gamma \log b + \varepsilon \log d$$

whence by subtraction

$$\log B_1 - \log B_2 = \alpha\,(\log w_1 - \log w_2),$$

or

(a) $$\alpha_{12} = \frac{\log B_1 - \log B_2}{\log w_1 - \log w_2} \quad (19)$$

α_{12} denotes that this particular value of α is derived from the related values of B_1, w_1, and B_2, w_2.

Again by using a constant difference in load, w, and *varying the lengths* l, we obtain a series of bendings B'_1, B'_2, B'_3, B'_4, B'_5, corresponding to the lengths l_1, l_2, l_3, l_4, l_5. Applying our general formula to the values of the bendings B'_1 and B'_2, for lengths l_1 and l_2, we have, in the same way as before,

$$\log B'_1 = \log C + \alpha \log w + \beta \log l_1 + \gamma \log b + \varepsilon \log d$$

$$\log B'_2 = \log C + \alpha \log w + \beta \log l_2 + \gamma \log b + \varepsilon \log d$$

from which

(b) $$\beta_{12} = \frac{\log B'_1 - \log B'_2}{\log l_1 - \log l_2}.$$

Similarly by determining the bending for a definite

weight in bars of the same metal, and having l and d the same, but *varying the breadth b*, we find for the successive breadths b_1, b_2, &c., the corresponding bendings B''_1, B''_2, &c., and deduce the relation

$$\text{(c)} \qquad \gamma_{12} = \frac{\log B''_1 - \log B''_2}{\log b_1 - \log b_2},$$

and finally by *varying the depth*, we have

$$\text{(d)} \qquad \varepsilon_{12} = \frac{\log B'''_1 - \log B'''_2}{\log d_1 - \log d_2}.$$

The apparatus (Fig. 16), consist of a graduated scale upon which slide two knife-edge supports for the metal bar, one of which is connected to a battery. At the middle of the graduated scale is mounted a micrometer screw

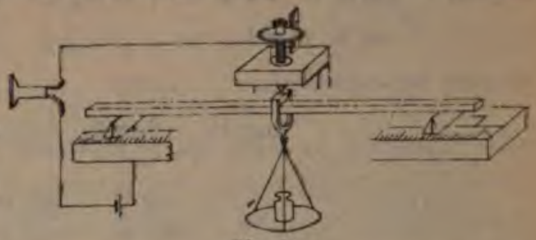

Fig. 16.

whose point rests upon the back of a knife-edged stirrup, sitting upon and at right angles to the bar at its middle point, and carrying the pan in which the weights are placed. The micrometer screw is connected to the other pole of the battery and the circuit is completed through the bar itself and the stirrup. A telephone receiver placed in the battery circuit gives notice of contact between the point of the screw and the stirrup.

In use the supports are placed at equal distances on each side of the micrometer screw, and the bar, with the stirrup on it, is so placed upon the supports as to leave equal lengths projecting over the supports at each end. The bar is brought directly under the screw point and the stirrup placed accurately at right angles to the bar and

under the screw. The pan containing the weights must hang clear of all objects. The apparatus having been perfectly adjusted the length of the bar l, between the supports, is read and recorded. The micrometer screw is then slowly turned down until the telephone gives notice of contact between the screw and the stirrup. The screw is then carefully turned back until the snap due to breaking the circuit is heard in the telephone. The zero reading is then taken, using the mean of three is each case.

For this experiment four bars of brass of rectangular cross-section and about 70 cm. in length are employed. One of the transverse dimensions of each bar is about 6 millimeters, so that when placed with this dimension vertical, we have a series of bars of constant depth. The other transverse dimension, for bars No. 1, 2, 3 and 4, is 10, 8, 6 and 3 millimeters respectively, so that when this dimension is placed vertical we have a series of bars of *constant breadth and variable depth.*

To determine the bending due to 100 grams, or to any weight whatever, it is better to take the zero reading with an initial weight in the pan. Then to find the bending for any given load, as 100 grams, the bending is taken for the initial load *plus the 100 grams, and the difference between this reading and the initial reading gives the bending for 100 grams.* A new initial load is taken and the same process is repeated, thus giving a second value for the bending for 100 grams; a third initial load is chosen and a third value for B is determined. The mean of the three values thus found is the bending for 100 grams. *This method is to be pursued in finding the bending for any weight whatever.*

FORM OF RECORD.

Bending for 100 grams. Date ....
Bar No. .... Length ....

Load.	Readings.	Means.	Difference for 100 grams.
50 g.	....	....	
150 g.	....	....	....
75 g.	....	....	
175 g.	....	....	....
100 g.		....	
200 g.		....	....
	Mean bending —		*B* for 100 g. ....

The exercise may now be completed under the following heads:

I. Vary w and determine B for five different weights. Use as loads the weights 75, 100, 150, 200, and 250 grams. Use bar No. 1, length 60 cms., laid flat-wise. Do not at any time exceed 300 grams. Determine B_1, B_2, B_3, B_4, B_5, for w_1, w_2, w_3, w_4, w_5. Apply formula (a), combining the observations two by two. The mean of the ten values of a obtained in this way is taken as the value of a to be substituted in the general formula. In case any value of B, as B_n, has been wrongly determined, then every value of a containing the subscript n will differ sharply from the rest and will indicate that the nth observation should be taken with greater care. A glance at the tabulated values of a will generally reveal any such pronounced error of observation.

FORM OF RECORD.

Exercise 14. Verification of the Laws of Bending.

First part as given on page 52. **Date** ....

Second part thus:

B	cms.	log B	w	grams	log w
B_1	....	.....	w_1	....	.....
B_2	....	.....	w_2	....	.....

and so on.

* * *

$\log B_1 - \log B_2 = \ldots.$	$\log w_1 - \log w_2 = \ldots.$	$\alpha_{12} = \ldots.$
$\log B_1 - \log B_3 = \ldots.$	$\log w_1 - \log w_3 = \ldots.$	$\alpha_{13} = \ldots.$

Mean value found for $\alpha = \ldots.$

II. Vary l and determine B′ for a constant load w = 100 grams. Use bar No. 2, laid flat-wise. For lengths 30, 40, 45, 50, 55 cms., determine B'_1, B'_2, B'_3, B'_4, B'_5 for the constant load w = 100 grams. Apply formula (b), combining the observations as in I. Take as the final value for β the mean of the ten values found as above. Record as in I, substituting l for w.

III. Vary b and determine B″, for a constant load of 150 grams. Use bars 1, 2, 3, 4, with constant depth. Measure b_1, b_2, b_3, b_4, by means of the micrometer gauge. Determine for b_1, b_2, b_3, b_4, the corresponding values B''_1, B''_2, B''_3, B''_4. Apply formula (c), computing as in I and II. Record as before.

IV. Vary d and determined B for a constant load, W = 150 grams. Use bars 1, 2, 3, 4, with constant breadth. For depths d_1, d_2, d_3, d_4, determine the bending B'''_1, B'''_2, B'''_3, B'''_4. Apply formula (d) in computing ε. Record as before.

V. Insert final values of α, β, γ, ε, in the general formula. Formulate the laws of bending in words.

Curves: To represent the first law by a curve, plot B on the axis of ordinates and w^a on the axis of abscissae. Similarly for the second law plot the values of B′ as ordinates and the related values of l'^3 as abscissae. Both curves should be straight lines.

Exercise 15. Young's Modulus by Flexure. As we have learned in the last exercise, the bending of a bar of length l, breadth b, and depth d, under a load w, is expressed by the equation

$$B = C \frac{w\,l^3}{b\,d^3}.$$

It has already been observed that the constant C, depends upon the mode of support and the material of which the beam is composed. It is shown in the theory of elasticity that when the beam is supported by its ends the value of the constant relating to its mode of support, is 1/4, and when supported from one end its value is 4. There remains therefore, the undetermined part of our constant C, depending upon the material of the beam. It may be shown from mathematical analysis that this part of the constant is 1/M, where M is Young's modulus for the material in question. If the bar is supported at the ends as usual, then we may write

$$B = \frac{w\,l^3}{4\,M\,b\,d^3}$$

or

$$M = \frac{w\,l^3}{4\,b\,d^3\,B}. \; *$$

---

* It may be shown directly that the expression for M as given above, is true for all bars supported at the ends and loaded at the middle. Such proof would, however, exceed the limits set for this text. See Stewart and Gee, vol. I, pp. 162-195.

If now we insert in this formula the dimensions of bar No. 1, we shall find for any bending B and the related load w, a value for M very nearly that obtained in exercise 13 as Young's modulus for brass.

Moreover it appears that Young's modulus is concerned here if we consider attentively what takes place in the bending of a bar. We see that upon the under side the bar is stretched, while upon the upper side a compression must ensue. Now the resistance to this stretching stress on the one side and to the compressing stress on the other, form the two parts of a couple tending to right the bar under its load, and the stress per unit area, divided by the strain per unit length gives again Young's modulus for the material in question.

FORM OF RECORD.

Exercise 15. To determine Young's modulus of brass and steel by the method of flexure.

I. For brass. Date............

From exercise 14, I.

w = ...	log w = ...
l = ...	log l^3 = ...
b = ...	colog 4 = ...
d = ...	colog b = ...
B = ...	colog d^3 = ...
	colog B = ...
M = ...	
	log M = ...

II. For steel.
(a) First part as on page 52.
(b) Computation as above.

Exercise 16. Coefficient of Simple Rigidity. As shown on page 40, the expression for the simple rigidity of a cylinder under torsional stress is

$$n = \frac{2 L \mathcal{T}}{\pi \theta r^4}. \qquad (7)$$

where $\mathcal{T}$ is the moment of the torsional couple needed to

produce an angular twist of θ radians in a circular cylinder of length L and radius r. In order to measure n we must determine the four quantities $\mathcal{T}$, θ, L, and r. This is most readily done by means of the following apparatus.

A metal rod, some 150 to 200 cms. in length, is supported horizontally in a frame so that it turns freely about its own axis. Near one end of the rod is clamped a short lever of length l, carrying at its outer end a pan for weights. The other end of the rod slips into a square socket attached to a vernier arm moving over a scale graduated to degrees, by means of which the angular twist of the rod may be determined. The vernier arm is supplied with a slow motion tangent screw, for adjustment before reading. The lever l, carries a stout wire ending in a sharp point, so arranged as to move in front of a short arm furnished with a similar point. By means of the tangent screw the two points may be brought opposite each other with considerable accuracy. The rod is to be brought to this position each time before a reading is made.

The manipulation is as follows : a zero load of 50 grams is placed in the pan, the rod brought up to position and the zero reading taken. A load of 100 gms. is then added, the rod again brought to position by the tangent screw and a second reading taken. The difference between these two readings gives the angular twist for 100 gms. The load in the pan is again increased by 100 gms. and the readings made. Half the difference between this reading and the first is likewise the twist for 100 gms. The load in the pan is increased in this way by successive steps of 100 gms. until 500 gms. have been added ; then diminished by similar steps until the zero load remains, the twist for 100 gms. being computed from each reading in combination with

the first. The mean of the values thus found is the angular twist θ, for a load of 100 grams. The quantities L and l are to be determined by measurement. Determine the radius of the rod from five measurements of the diameter by means of the micrometer gauge.

FORM OF RECORD.

Exercise 16. To determine the coefficient of simple rigidity of two metals.

Date............

Load.	Readings.	Diff. for 100 gms.
50 gms.	......	....
150 "	......	....
250 "	......	....
350 "	......	....
		Mean
		....

Computation:

$$n = \frac{360 \cdot L \cdot m \cdot g \cdot l}{\pi^2 \theta r^4}$$

	log 360 = ...
L = ...	log L = ...
m = 100	log m = ...
g = 980	log g = ...
l = ...	log l = ...
θ = ...	colog g = ...
r = ...	colog π^2 = ...
	colog r^4 = ...
	log n = ...
	n = ...

Exercise 17. Coefficient of Simple Rigidity of Brass Wire from Torsional Vibrations. We have learned from Hooke's law that, within the limits of elasticity, the restoring force called out by any distortion is simply proportional to that distortion. An important consequence of this law is, that if a heavy body be suspended by a wire and the wire be twisted through a moderate angle and then released, the restoring force is continually proportional to the distortion. The motion induced is simple harmonic and consequently the vibrations of the body are isochronous. Now if I be the moment of inertia of the body, and $\mathcal{T}_1$ the moment of the torsional couple produced by unit angular twist, then T,

the time of a complete vibration of the system, is given by the equation

$$T = 2\pi \sqrt{\frac{I}{\mathcal{T}_1}} \tag{21}$$

from which

$$\mathcal{T}_1 = \frac{4\pi^2 I}{T^2}. \tag{22}$$

We have also seen on page 40, that the coefficient of simple rigidity is defined by the equation

$$\mathcal{T}_1 = \frac{\pi n r^4}{2L} \tag{8}$$

whence by equating values for $\mathcal{T}_1$, and solving for n, we have

$$n = \frac{8\pi I L}{r^4 T^2}, \tag{23}$$

an expression involving only quantities amenable to measurement.

The apparatus consists of a heavy lead cylinder suspended by a brass wire and furnished with a light pointer moving over a horizontal circular scale. By means of an adjustable clamp the length of the torsional pendulum may be varied within wide limits. The times of vibration T_1, T_2, T_3, for lengths L_1, L_2, L_3, are determined to thousandths of a second by the method given under Time, page 28. The moment of inertia I, of the cylinder is computed from its mass M, and radius R. The radius of the wire is determined by means of the micrometer gauge. The insertion of any pair of related values of T and L in formula (23) gives a value for n. Compute the three values and return the mean as the value found for n.

FORM OF RECORD.

Exercise 17. Coefficient of simple rigidity of brass wire from torsional vibrations. Date............

First part as given under Exercise 6.

Second part thus:

L_1 cm....	T_1 sec....		
L_2	T_2		
L_3	T_3		
	M	= 10105.8	gm.
	R	=	cm.
	I	=	gm. cm^2.
	r	=	cm.

Computation:

$$n = \frac{8\,\pi\,I\,L}{r^4\,T^2}$$

log 8	= ....
log π	= ....
log I	= ....
log L	= ....
colog r^4	= ....
colog T^2	= ....
log n	= ....
n	= ....

CHAPTER IV.

PENDULUM EXPERIMENTS AND MOMENT OF INERTIA.

Exercise 18. The Simple Pendulum. The object of this exercise is to investigate the relation between the time of a simple pendulum and its length. Since the most casual observation shows that the period of a pendulum is some function of the length, we may assume as a general expression for the existing relation

$$T = C\, l^m \tag{24}$$

where C and m are constants to be determined by experiment. Passing to logarithms and solving for m, as in exercise 14, we find for m the value

$$m_{12} = \frac{\log T_2 - \log T_1}{\log l_2 - \log l_1}. \tag{25}$$

From a series of observations on five pendulums of different lengths we get, by combining as in exercise 14, ten values of m. The mean value of m so determined, when inserted in the equations connecting related times and lengths, gives five independent equations for C, of the form $\log C = \log T - m \log l$. The mean value of C thus determined, and the mean value of m when inserted in

equation (24), give the relation sought. The exercise is to be completed as follows:

(a) Determine to thousandths of a second the period of vibration of a pendulum for five lengths 120, 140, 160, 180 and 200 cms.

(b) Compute from these observations the values of m and C as described above, observing the arrangement adopted on page (53).

(c) Plot curve, using values of log T as ordinates and those of log l as abscissae.

FORM OF RECORD.

Exercise 18. To determine the law of the simple pendulum.

Date ....

(a) Use form of record given under Exercise 6.

(b)

l	log l	T	log T	m	C

Exercise 19. Computation of g. From the well known formula for the time of vibration of a simple pendulum

$$T = 2\pi \sqrt{\frac{l}{g}} \tag{26}$$

we see that

$$C = \frac{2\pi}{\sqrt{g}} \tag{27}$$

or

$$g = \frac{4\pi^2}{C^2}.$$

From the mean value of C found in experiment 18, compute the value of g.

FORM OF RECORD.

Exercise 19. To compute value of g.

Date............

C =

log C =

log 4 = ...

log π^2 = ...

colog C^2 = ...

log g = ... g = ...

Exercise 20. To find the Moment of Inertia of an Iron Bar.

Equation (21) defines the moment of inertia of a body performing torsional vibrations, in terms of the period of vibration T, and the moment of the torsional couple $\mathcal{T}_1$, tending to produce rotation. Suppose the vibrating body to be a rectangular bar of moment of inertia I. If suspended so as to swing freely about a vertical axis by a stout wire it will tend to return to its position of rest if disturbed. Let the moment of this restoring couple, produced by unit twist be $\mathcal{T}_1$. Then

$$T = 2\pi\sqrt{\frac{I}{\mathcal{T}_1}} \tag{21}$$

If now there be added to the bar a ring whose moment of inertia I_r, may be readily calculated from its dimensions, (see Table V), then the period of the system becomes

$$T_1 = 2\pi\sqrt{\frac{I + I_r}{\mathcal{T}_1}} \tag{28}$$

Eliminating $\mathcal{T}_1$ from these two equations we have

$$I = I_r \cdot \frac{T^2}{T_1^2 - T^2} \cdot \tag{29}$$

Again if to the original system there be added a pair of cylinders, each of mass m, and radius r, symmetrically placed near the ends of the bar, at distances a, from the axis of rotation, the period of the combined system becomes

$$T_2 = 2\pi \sqrt{\frac{I + 2I_c}{\mathcal{T}_1}} \qquad (30)$$

where I_c, the moment of inertia of a single cylinder about the axis of rotation, is given by the equation

$$I_c = \frac{m\,r^2}{2} + m\,a^2 \;\; * \qquad (31)$$

Combining equations (21) and (30) we find

$$I = 2I_c \cdot \frac{T^2}{T_2^2 - T^2} \,. \qquad (32)$$

Finally, if to the original system we add both ring and cylinders and determine the period of vibration of the system T_3, we secure a third value for I in the same way as above, from the equation

$$I = (I_r + 2I_c) \cdot \frac{T^2}{T_3^2 - T^2} \,.$$

The apparatus consists of a flat rectangular bar about 14 cms. long and 2 cms. wide, to which is attached at its middle point and normal to its plane, a small rod furnished with a hook at its upper end, by means of which the bar may be suspended by a stout phosphor-bronze wire. The bar carries at one end a light paper pointer which serves to mark the transits of the vibrating system across the position of rest. A large flat iron ring having its sides and edges well polished and a diameter marked upon its outer edges, is next placed upon the bar with its axis coincident with the axis of rotation. The proper position of the ring upon the bar is assured by means of two arcs of circles struck upon the upper surface of the bar, with which the outer edges of the ring must coincide, while the ends of the diameter must lie in a line drawn length-wise through the middle of the bar.

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* See Carhart's University Physics, Vol. I, p. 91.

Two small, accurately turned iron cylinders are also provided, to be placed upon the small circles at the ends of the bar, their centers being distant by a length a, from the center of the bar and axis of rotation.

The exercise comprises the following measurements:

(a) Measure the external and internal diameters of the large ring by means of the vernier caliper and compute the external and internal radii, r_1 and r_2.

(b) Measure the distance between the centers of the two circles at the ends of the bar and compute the length a, from the axis of rotation of the system to the center of either cylinder when placed on the bar.

(c) Measure the diameter of the cylinders and compute the mean radius r.

(d) Determine the mass of large ring M_r, and of either of the two cylinders M_c.

(e) Determine to thousands of a second the time of vibration of the bar alone, T; of the bar and ring, T_1; of the bar and cylinders, T_2; of the bar, ring and cylinders, T_3.

FORM OF RECORD.

Exercise 20. To determine the moment of inertia of an iron bar.

(a) Enter results as in exercise 6.

(b)

Value	log	Value	log
M_r		T	
r_1		T_1	
r_2		T_2	
I_r		T_3	
M_c		$T_1 + T$	
r_c		$T_1 - T$	
a		....	
I_c		....	
		....	

Computation:

$$(29)\quad I = I_r \frac{T^2}{T_1^2 - T^2}$$

$\log M_r =$
$\log (r_1^2 + r_2^2) =$
$\text{colog } 2 =$
$\log I_r =$
$\log T^2 =$
$\text{colog } (T_1 + T) =$
$\text{colog } (T_1 - T) =$
$\log I =$
$I =$

CHAPTER V.

OPTICAL MEASUREMENTS.

RADIUS OF CURVATURE.

It is shown in treatises on optics that the effect of a mirror or of a lens of any form, consists in impressing upon the wave-front of the luminous disturbance a curvature directly related to the curvature of the mirror or lens in question. By definition the curvature at any point in a curve is the reciprocal of the radius of the osculating circle at that point. Since the effects produced by mirrors or lenses are to be predicted from a knowledge of their constants, it becomes a matter of importance to measure the curvature of an optical surface, in other words to determine its radius of curvature.

The surface most commonly employed in optical construction is that of the sphere, since only the largest mirrors or lenses possess surfaces differing noticeably therefrom. Concave or convex mirrors may therefore be regarded as parts of spherical shells, with the inner or outer surface polished as the case may be. The radius of curvature of such a mirror is obviously the radius of the sphere, of which the mirror forms a part. In lenses both surfaces are to be regarded as parts of spheres of definite

radii. In the case of a plane surface the radius is of course infinite.

Exercise 21. Radius of Curvature of a Lens by the Spherometer.

The experiment consists in determining the radius of curvature from a careful measurement of the amount by which the lens surface departs from a plane, i. e., by measuring the "sagitta."* If we place a spherometer upon a lens with the three feet resting upon the surface of the lens, we may conceive a plane passed through the lens, cutting from it a segment whose base is a circle passing through the three feet of the instrument. At right angles to the base of this segment stands the micrometer screw of the spherometer, and by taking readings, first upon a plane surface and then upon the lens, the sagitta of the curve, that is the distance the central foot of the instrument is above or below the plane containing the other three feet, may be accurately determined. Thus in Fig. 17, let A C B D A represent a vertical section of the segment, A C B the curved surface of the segment, and A B the diameter of its circular base. Then CD = s, is the sagitta, DB = d, the distance from the center of the instrument to either of the three legs, and CO = BO = r, the radius of the sphere of which the lens surface is a part. By geometry

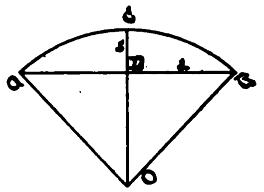

Fig. 17.

$$CD\,(2r - CD) = BD^2$$

or, inserting values

$$s\,(2r - s) = d^2 \qquad (33)$$

* See Preston's Theory of Light, p. 80.

whence

$$r = \frac{d^2 + s^2}{2\,s}. \qquad (34)$$

The distance d is usually measured once for all on the dividing engine or comparator, and is called the constant of the instrument. The distance d may also be determined in terms of the length of the side of the equilateral triangle formed by the feet of the spherometer as follows: Press the inment firmly upon a piece of stiff paper until the positions of the three feet are left sharply defined. The length of the sides of the triangle may then be accurately measured by the vernier caliper. Then from Fig. 18,

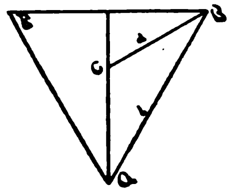

Fig. 18.

$$d^2 - \frac{d^2}{4} = \frac{l^2}{4} \text{ or } d = \frac{l^2}{3},$$

whence by substitution

$$r = \frac{l^2}{6\,s} + \frac{s}{2}. \qquad (35)$$

In practice the spherometer is first placed on a piece of plate glass and the zero reading accurately determined. It is then transferred to the lens and the readings upon the lens are made, care being taken to prevent the feet from slipping off the lens. The difference between the zero and the final readings gives the value of s. From the known value of d, the value of r is at once computed, or the value of l may be determined as shown above and the value of r computed from equation (35).

FORM OF RECORD.

Exercise 21. To determine the radii of curvature of lenses 1, 2, 3, 7, and 9, by the spherometer.

d .... l - .... Date ....

Zero	Lens 1.	Lens 2.	Lens 3.	Lens 7.	Lens 9.
......	......	......	......	......	......
......	......	......	......	......	......
......	......	......	......	......	......
Mean	Mean	Mean	Mean	Mean	Mean
	s	s	s	s	s
	r	r	r	r	r

Measure both sides of each lens.

Exercise 22. Focal Length of Lenses.

From the well known formula for the focal length of a lens.

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}, \qquad (36)$$

we may deduce an important relation under the condition that the object and image remain at a fixed distance, greater than 4f, from each other. Let l be the distance between the object and the screen upon which the image is received. Then there will be two positions of the lens for which a sharp image is projected upon the screen, one near the object giving an enlarged image, and another nearer the screen giving a small but bright image. Let a be the distance between these two positions of the lens. Then

$$p + q = l, \text{ and } q - p = a.$$

whence

$$q = \frac{l + a}{2} \text{ and } p = \frac{l - a}{2}$$

substituting in (36)

$$f = \frac{l^2 - a^2}{4l}.* \qquad (37)$$

*Owing to the fact that the distances p and q are not measured from the same point, but from the two principal points of the lens, this formula is not strictly accurate; the error is, however not large. For the correction due to this approximation, see Glazebrook and Shaw, Practical Physics, p. 350.

The apparatus consists of an optical bench about two meters long, provided with a scale reading to millimeters and two supports to carry the screen and the lens. The object, usually a pin or the hand of a watch, is placed at the zero end of the scale and at a suitable height above it. This is strongly illuminated by a lamp placed close behind it so that the brightest part of the flame, the object, the center of the lens and the middle of the screen are all in the same straight line. A rough approximation to the value of f may be obtained by placing a piece of white paper in front of the object and bringing the lens toward it, until there is formed upon the paper an image of the window bars opposite, or of the trees and buildings outside. The reading of the lens carrier gives at once the approximate focal length. Why is not this the true value of f? The screen should then be placed at a distance from the object not less than five nor more than seven times this rough value of f. (Why?)

The lens is now shifted until a sharp image is projected upon the screen. The mean of five settings is taken as the position of the lens. The second position of the lens for a sharp image is then determined in the same way. The difference between these mean values is a, and this value with its related value of the setting of the screen l, will, when substituted in the formula, give a value for f. At least three different settings of the screen should be used and the mean of the three values of f returned as the focal length of the lens.

In the case of a concave lens, there can, of course, be no real image. Therefore, in order to use this method, it is necessary to combine the concave lens with a convex lens of suitable curvature, and determine first the focal

length of the combination and then the focal length of the convex lens separately. If F be the focal length of the combination, and f′ that of the auxiliary lens then the focal length of the concave lens, f, is given by the relation

$$\frac{1}{f} = \frac{1}{f'} - \frac{1}{F}$$

or

$$f = \frac{F\ f'}{F - f'} . \qquad (38)$$

Use lens 10 as auxiliary lens in case a concave lens is found.

FORM OF RECORD.

Exercise 22. To determine the focal lengths of lenses 1, 2, 3, 7, and 9.

Date............

Lens No.	Screen (l)	Lens 1st position	Lens 2nd position	a	$l^2 - a^2$	4 l	f
1.	......	....	....	...	....	....	....
	......	....	....	...	....	....	....
	......	....	....	...	....	....	....

Focal length of lens 1 = ...

Exercise 23. Index of Refraction of Lenses from Radii of Curvature and Focal Lengths.

It is shown in geometrical optics that the focal length of a lens for any wave length is a function of the index of refraction μ, of the glass for light of that wave length, and of the radii of curvature of the lens. This relation is given by the equation

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) . * \qquad (39)$$

If we have the values of f, r_1 and r_2 for any lens we may compute the index of refraction at once from the

*For the derivation of this formula and its interpretation see Carhart's University Physics, Vol. I, pp. 271-277.

above formula. The focal length having been obtained by means of white light, the resulting value of μ will of course refer to no definite color, but will in general correspond to the brightest part of the spectrum, i. e., to the part between the lines D and E.

FORM OF RECORD.

Exercise 23. From the values of f, r_1 and r_2 for the lenses 1, 2, 3, 7, and 9, compute the mean index of refraction for each lens.

Date ....

Lens.	r_1	r_2	f	μ
1	....	....	....	....
2	....	....	....	

How may equation (39) be simplified, when one of the radii is infinite? When the two radii are equal?

Exercise. 24. Lens Curves.

We have seen from Exercise 22, that for every setting of the screen there are in general two positions of the lens for which a sharp image is obtained. If now we plot the settings of the screen as ordinates and the corresponding settings of the lens as abscissae, we obtain what is known as the "lens curve" Fig. 19. From our nomenclature the equation to the curve is

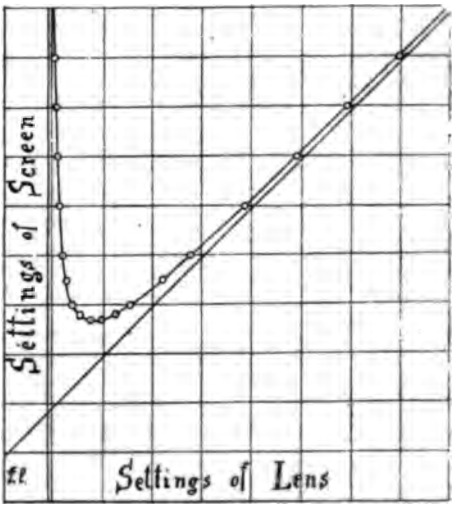

Fig. 19.

$$\frac{1}{x} + \frac{1}{y-x} = \frac{1}{f}. \qquad (40)$$

Is this the equation to an hyperbola? If so, what are

its asymptotes? What is the physical interpretation of each? Use lens 10 for this experiment. Determine at least fifteen separate positions of the screen with their related settings of the lens. Plot the curve and draw the asymptotes. Take pains to obtain as many as four or five points near the bend of the curve, i. e., where the two images approach each other. What is the value of y for the lowest point of the curve? Determine the focal length of the lens from the curve.

FORM OF RECORD.

Exercise 24. Lens curves.

Lens.......... . Date..........

Screen (y).	Lens (x_1).	Lens (x_2).
......	......	......
......	......	......
......	......	......

Focal length = ... Plot curve.

Exercise 25. Magnifying power of the Telescope.

The telescope in its simplest form consists of two lenses, the object-glass or objective L, a convex lens of long focus, and the eye piece L′, a short focus lens either convex or concave.

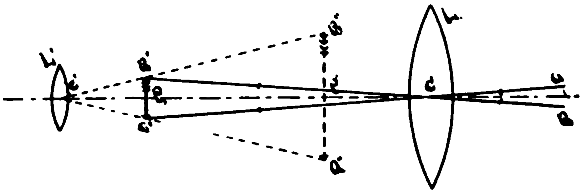

Fig. 20.

The distance from the object to the instrument is always great as compared with the focal length of the objective and the image is consquently smaller than the object in all cases. In case the eye-piece is a convex lens, (Fig. 20), this small image is viewed directly by the eye-piece as an object placed nearer the lens than its focal distance. The result is a magnified virtual image of the image.

The effect of a telescope is to increase the visual angle subtended by a very distant object, that is, to bring an image of the object near the eye, so that when this image is viewed by the eye directly, the visual angle subtended by it is larger than that subtended by the object, in the ratio $\frac{F}{25}$, where F is the focal length of the objective and 25 cms. represents the distance of distinct vision for the normal eye. This relation is readily seen from Fig. 21, where the objective L, forms an image of a distant object upon a screen. An eye at the center of the objective would see both image and object as of the same size, since the angles subtended are equal. If however, the eye approach the screen, the angle subtended by the image will increase until at a distance of twenty-five cms. from the screen the image will appear larger than the object in the ratio $\frac{F}{25}$, as given above.

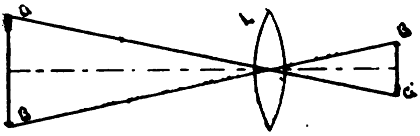

Fig. 21.

If the eye be brought nearer to the image in order to increase the magnification, its power must be increased by the use of a lens used as a simple magnifier. Such a lens is termed an eye-piece. The magnification produced by the eye-piece is $\frac{25}{f}$ where the focal length of the eye-piece is f. The total magnification of the two lenses forming the telescope is therefore the product of the two, or $\frac{F}{f}$. This ratio is called the magnifying power of the telescope, and is most readily measured as follows:

A long scale is set up at one end of the room and so lighted that the divisions shall be seen sharp and clear. The telescope is focussed upon the scale so as to give a sharp image. *The observer next looks through the telescope with the right eye and views the scale directly with the left eye.* A little adjustment of the direction of the telescope and a little patience will enable the observer to see the two images formed by the two eyes, overlapping, so that he sees at the same time (Fig. 22), the complete scale, and projected upon it, the magnified divisions of the scale itself. By careful adjustment of the telescope the lengths of these magnified divisions may be read directly in terms of the divisions of the scale. Thus, suppose the half division from 4 to $4\frac{1}{2}$ is seen projected upon the scale, its upper edge appearing to be at 4.1 and its lower edge at 8.15. It is clear that one-half division seems to cover 4.05 divisions, hence the magnifying power is 8.1.

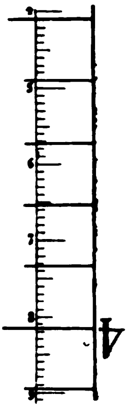

Fig. 22.

Care should be taken to avoid touching the telescope or its support during the measurements, as well as to avoid moving the head while comparing the upper and lower edges of the image for coincidence with the scale divisions. Measure the magnifying power of the telescope at distances of 4, 7, 10, and 15 meters from the scale. Next remove the field combination, by unscrewing the telescope at the first joint from the eye-piece, and taking out the lens found there. Repeat the measurement as above. What is the purpose of the field combination? How does the magnification vary with the distance?

FORM OF RECORD.

Exercise 25. To determine the magnifying power of a telescope.

Date ....

Distance from scale.	Magnifying power with combination.	Magnifying power without combination.
. . ..,......	. .. .	. . . . .
. ...	....	. . . . . .

Effect of distance upon magnifying power.

Exercise 26. Radius of Curvature by Reflection.

The radius of curvature of a polished spherical surface may be determined by means of purely optical considerations if we employ the phenomena and formulæ relating to spherical mirrors. Assume that the convex spherical surface m' (Fig. 23), is placed before the telescope T, at a distance A, and that it receives light from two brilliant objects L and L' symmetrically placed with respect to T. There will be formed in the mirror m', two virtual, erect and diminished images of the objects L and L'. Owing to the inversion of these images by the telescope they are seen inverted in T. A small scale ss', placed in contact with the lens enables the observer to read off directly the apparent distance ss' between the two images. Now since the rays from L and L', after reflection at s and s' enter the telescope and seem to come from the images l and l', the normals Cs and Cs' will, if produced, bisect approximately the angles LsT and L's' T, and to the same degree of approximation, PQ = 1/2 LL' where P and Q are respectively the intersections on LL' of Cs and Cs' produced.

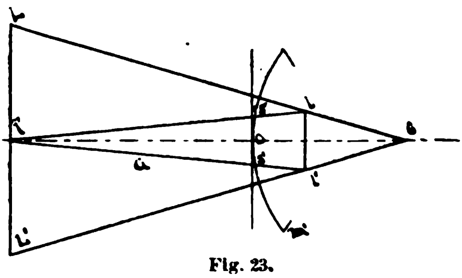

Fig. 23.

Let ss' = s; OT, the distance from the lens to the objective of telescope, = A; OC = R; and LL' = L. Then from the triangles PQC and ss'C we have

$$\frac{L/2}{s} = \frac{R + A}{R},$$

or

$$R = \frac{2\,A\,s}{L - 2s}. \qquad (41)$$

In practice the lens is held in a clamp supported upon a tripod base, one foot of which bears an adjusting screw for tilting the lens about a horizontal axis. This foot should stand in a line parallel to the axis of the telescope, and normal to the lens surface. Two small lamps are placed at L and L' with their flames turned edge-wise to the lens. The telescope and lens are set up on two tables at least three meters apart, the lens facing the most brightly lighted window in the room. The telescope is focussed upon the lens surface until the scale ss' is sharply defined. One observer then takes one of the lamps and moves it slowly back and forth and up and down along the line TL, until the other catches sight of the moving image in the telescope.

It is to be noted that the image in a convex mirror is erect and is seen inverted owing to the inversion in the telescope; this inversion applies to the motions of the lamp as well, so that if the light moves to the right, the image seen in the telescope moves to the left and *vice versa.* Should the image fail to appear when the above directions are followed, the lens holder should be rotated slightly about its vertical axis until the image appears in the field. The image is then brought to the level of the scale by means of the adjusting screw in the foot of the lens holder.

A black cloth placed close behind the lens renders the image much more bright and distinct.

Care must be taken to avoid confusion of the true images from the front surface of the lens, with the pair of erect images seen in the telescope which are due to reflection from the back of the lens; these images are originally inverted owing to the concave surface, and are erected by the telescope. Which pair of images must be chosen in case of a concave lens? What change is needed in the formula?

It will usually be found necessary to change the focus of the telescope very slightly in order to fix sharply the position of the image on the scale. This difference in focus becomes the more marked the more nearly the lens surface approaches a plane. The method is therefore best adapted to lenses of large curvature. The small scale may be dispensed with by pasting upon the lens two strips of paper with straight edges, parallel and facing each other. the perpendicular distance between the edges of the strips is then carefully measured with the vernier caliper and recorded. The lamps are then so adjusted that their respective images just disappear behind the edges of the paper. The measured distance is then equal to s. The distances A and L should be measured with a steel tape or a long stick and a metric rule.

Measure by this method the radii of curvature of lenses 1, 3, and 9.

FORM OF RECORD.

Exercise 26. To determine the radii of curvature of lenses 1, 3, and 9, by method of reflection.

Date ....

Lens No.	A	L	s	R
1	....	....	....	....
3	....	....	....	....
9	....	....	....	....

Exercise 27. Index of Refraction by Means of a Microscope.

It is shown in works on physics * that if a point A, (Fig. 24), be viewed vertically through a transparent plate of thickness OA, and refractive index μ, the point will appear to be raised to some position I in the vertical, such that OA = μ.OI, or

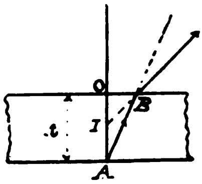

Fig. 24.

$$\mu = \frac{OA}{OI} = \frac{t}{t-a}, \qquad (42)$$

where AI = a.

In this way the index of refraction of a transparent plate, or of a layer of fluid may be determined by means of a microscope furnished with a scale and vernier on its tube.

In practice the microscope, fitted with a low power objective is focussed upon a mark on a piece of stiff paper, or better upon a scratch in a piece of flat metal, held upon the microscope stage by means of clips or bits of wax. The instrument having been sharply focussed upon some prominent feature of the scratch, the position is taken by reading the scale and vernier on the tube. The transparent plate, usually a plate of glass some 5 mm. thick, is next placed upon the stage *above and immediately in contact with* the scratch in the plate. The microscope is again focussed upon the same feature of the scratch *through* the plate, and the reading taken as before. The microscope is then focussed upon the upper surface of the plate and the reading made. From these three readings, each being the mean of at least five separate settings,

* See Carhart's University Physics. Vol. I. p. 261.

the values of OA and OI are readily determined and the value of μ computed from the formula.

For *liquids* a small flat bottomed dish is fastened to the microscope stage by two bits of wax, and the instrument focussed upon a scratch on the upper surface of the bottom. The liquid is added by means of a medicine dropper to a depth of from 3 to 5 mm., and the reading taken upon the *same scratch through the liquid.* A few grains of lycopodium powder are then sifted upon the surface of the liquid, the microscope focussed upon a grain of the floating powder and the reading taken as before. For liquids the instrument must of course stand vertical. In case the readings differ by as much as 0.06 mm., the mean of a larger number of readings must be taken. The depth of the liquid may be increased after each determination, and readings *through* the liquid and *on top* of the liquid give data for a new value of μ.

Determine by this method the refractive indices of two pieces of glass and of distilled water.

FORM OF RECORD.

Exercise 27. To determine the index of refraction of glass and water by means of a microscope.

Date............

Reading on scratch.	Through subst.	On top	t	t − a	μ
......	....	....	..	....	..
......	....	....	..	....	..
......	....	....	..	....	..

THE SPECTROMETER.

The spectrometer (Fig. 25), consists of a telescope

mounted so as to move freely about a vertical axis, a graduated circle concentric with the vertical axis and provided with verniers or reading microscopes for determining the exact position of the telescope

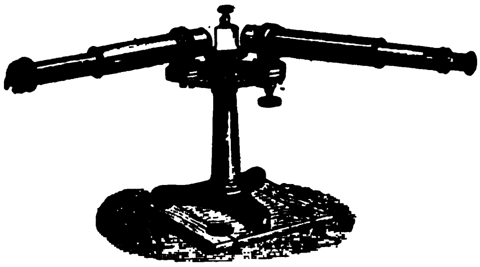

Fig. 25.

at any time, and a collimator, or tube carrying at its outer end an adjustable slit and at the inner end a double convex lens, by means of which all light proceeding from the slit may be made to leave the collimator as parallel rays. Upon the vertical axis is placed a small table to support the prism, grating, or crystal to be studied by means of the spectrometer.

Adjustments. A number of adjustments must be made before the spectrometer is ready for use.

(a) The eye-piece of the telescope has at its focus a pair of fine hairs termed cross-hairs, which must be sharply seen by the eye on looking into the telescope. If the cross-hairs are not sharp, the eye-piece must either be drawn out or pushed in with a gentle twisting motion until the cross-hairs are seen sharply defined on a white field when the telescope is turned toward the window.

(b) The telescope itself must be focussed for parallel rays. This is best done for the first time by removing the telescope from its carrier and focussing upon some very distant object, as the moons of Jupiter or the planet Venus.

(c) The telescope is then replaced and turned so as to look directly into the collimator. A small lamp is

placed behind the slit, which is slightly opened, and the outer end of the collimator either pulled out or pushed in until the slit is seen sharply focussed in the telescope. The images of slit and cross-hairs must show no parallax, that is, there must be no apparent motion of slit and cross-hairs with reference to each other as the head is slightly moved from side to side while looking into the telescope.

The collimator is now in adjustment, i. e., the slit is at the principal focus of the collimator objective, since the telescope focussed for parallel rays shows the slit sharply focussed, and the rays therefore, emerging from the collimator must be parallel. In practice it is best to mark this position of the collimator once for all, and having brought the collimator slit to the indicated position the telescope is focussed upon the slit directly, and adjustments b and c are made.

(d) The axes of the telescope and collimator must stand normal to the vertical axis of the instrument. The telescope and collimator may each be raised or lowered by means of adjusting screws, but *ordinarily the collimator is placed in adjustment by the instructor* and the telescope is raised or lowered until the image of the slit falls in the central part of the field. The telescope is clamped in position by means of a thumbscrew before readings are made, and care should be taken that the setting of the telescope is not changed thereby. In moving the telescope *take hold close up to the circle and not by the outer end.*

Exercise 28. To Measure the Angle of a Prism.

The adjustments having been made, place the prism upon the table with the angle to be measured immediately over the center of the small table, and facing the collimator

so as to divide the opening about equally. Turn the telescope to position T, (Fig. 26). A black cloth placed loosely over the collimator, prism and telescope aids materially in finding the reflected image of the slit in the telescope. It is best to catch the reflected image first in the eye placed close up to the prism and then, keeping the image in view slowly bring the telescope into position. Having found the image reflected from the right side of the prism turn the telescope to position T′, and see if the image from the left side is also visible and the telescope in such a position that readings may be made in each case.

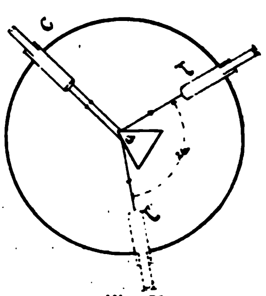

Fig. 26.

The slit is now brought down to a narrow line by means of the adjusting screw and the middle vertical cross-hair made to bisect the image of the vertical slit. The telescope is then clamped in position and the reading in position T′ made. The telescope is next unclamped, turned to position T, the image bisected by the cross-hair and the reading made. The difference between these two readings is twice the angle of the prism. Prove this. Displace the prism slightly and repeat the measurements three times.

FORM OF RECORD.

Exercise 28. To measure the angle of a prism by means of the spectrometer.

Date ....

Position T.	Position T′.	Difference.	Angle A.
......	....	....	....
......	....	....	....
......	....	....	....

Exercise 29. Index of Refraction of a Glass Prism.

The effect of a prism upon light passing through it is two-fold. The direction of the light is changed, the light being bent toward the base of the prism both on entering and leaving the prism, and secondly, the light is dispersed or broken up into its constituent colors. If the prism used in the previous exercise, be now placed centrally over the center of the table and turned into the position indicated by the full line (Fig. 27), an eye placed in the position indicated by the emergent light, will perceive no longer a bright image of the slit, but a broad band of color, the spectrum of the light furnished by the lamp. This spectrum may now be received into the telescope and its parts examined. The best effect is obtained by excluding all stray light from the telescope by means of the dark cloth as in Exercise 28. By rotating the prism slowly and following the spectrum with the telescope, a position is soon found in which, no matter which way the prism is rotated, *the spectrum comes to a certain point nearest the direct line from the collimator, stops and then recedes. This is the position of minimum deviation.* The small lamp is now removed and a Bunsen burner substituted. The burner is so arranged that the colorless flame plays against the tip of a piece of asbestos paper saturated with sodium nitrate. An intense yellow light results. On examining the image in the telescope it is seen that the spectrum of this light consists of a single bright line, a

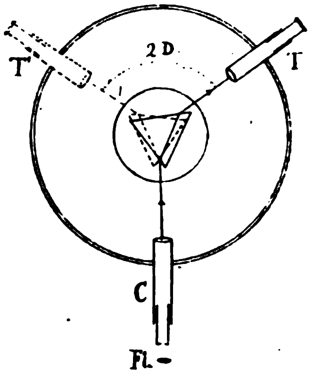

Fig. 27.

yellow image of the slit, the sodium spectrum, *for this temperature*. In spectroscopes of high resolving power this line is readily seen to consist of two lines, D_1 and D_2.

By closing the openings of the Bunsen burner so as to give the luminous flame, the continuous spectrum returns and we see superposed upon it the bright line due to the vapor of incandescent sodium. The non-luminous flame having been restored, the prism is rotated until the position of minimum deviation for the sodium line is accurately determined, the cross-hair placed upon the image of the slit, the telescope clamped and the reading taken.

The prism is next rotated into the position shown by the dotted line in the figure. The light is now deviated to the left of the direct position and the position of minimum deviation is determined as before. The difference between the two readings is obviously 2D, where *D is the angle of minimum deviation for sodium light*. It is shown in works on physics,* that when the prism is put in the position of minimum deviation, the refractive index μ, is defined by the equation

$$\mu = \frac{\sin 1/2\,(A + D)}{\sin 1/2\,A} \qquad (43)$$

where A is the angle of the prism. Derive this formula.

From the measured values of D and A as obtained above, compute the value of μ for sodium light for the prism under experiment. Repeat the experiment using lithium carbonate in place of sodic nitrate.

*Carhart. University Physics. Vol. I. p. 267.

FORM OF RECORD.

Exercise 29. To determine the index of refraction of a prism for the lines Na_a and Li_a.

Prism............ Date............

T	T'	2D	D
....	....	....	....
....	....	....	....

A	Computation.
D	log sin $\frac{1}{2}$(A + D) ......
A + D	log sin $\frac{1}{2}$ A ......
$\frac{1}{2}$(A + D)	log μ ......
$\frac{1}{2}$ A	μ ......

Exercise 30. Wave-length of Sodium Light by Diffraction Grating. A Bunsen burner (Fig. 28), is provided with a

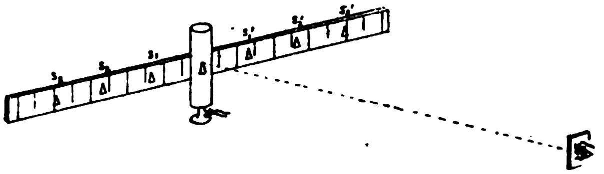

Fig. 28.

sheet iron hood in which is cut a small triangular slit, s, about 1.5 cms. long. Immediately in front and below the slit is fixed a meter rod held horizontally with the slit at the center of the rod. At a distance of some three or four meters in front of the slit is placed the grating, held in a suitable clamp, with its surface vertical and parallel to the meter rod. If the burner be adjusted for the luminous flame the slit appears white to the naked eye but when viewed through the grating the eye preceives in addition to the white slit, a number of spectra symmetrically placed with reference to the slit. These are diffraction spectra and are characterized (a) by the relative position of the various colors with respect to the slit, the violet being the

least diffracted and the red the most; (b) by the uniformity of the dispersion of the various spectra, each color being seen at a distance from the slit directly proportional to the wave-length of the light in question.

In practice the luminous flame is replaced by the sodium light and the colored spectra become a series of yellow images of the slit, which are seen by the eye placed behind the grating, projected upon the meter rod at spaces equidistant from the central image of the slit. Beginning at the inner spectra measure carefully the distances $s_1s'_1$, between the first two images on either side of the slit, s_2s_2', the distance between the next two, and so on. Take half the measured distance as the distance of each image from the central slit, ss_1, ss_2, and so on.

If the grating space be d, n the order of the spectrum observed, and θ the angle subtended at the eye by the distance ss_n, then

$$n\lambda = d \sin \theta_n \text{ *} \qquad (44)$$

where λ is the wave-length of sodium light. Hence for the first three or four spectra,

$$\lambda = d \sin \theta_1 = \frac{d \sin \theta_2}{2} = \frac{d \sin \theta_3}{3}, \text{ etc.}$$

In the experiment described the distances ss_n divided by a, the distance from the slit to the grating give directly $\tan \theta_n$, in each case, from which the value of $\sin \theta_n$ is readily found.

Determine by this method the wave-length of sodium light, using spectra of at least four different orders. In the grating used the value of d will be given by the instructor. Return λ in millimeters.

---

* Carhart. University Physics. Vol. I. p. 304.

FORM OF RECORD.

Exercise 30. To measure the wave-length of sodium light by diffraction grating.

d = ... s = ... a = ... Date............

s_n	s'_n	ss_n	$\frac{ss_n}{a}$	$\frac{\sin\theta_n}{n}$	λ
....	....	....	....	....	....
....	....	....	....	....	....
....	....	....	....	....	....
....	....	....	....	....	....

CHAPTER VI.

MEASUREMENTS IN HEAT.

Heat phenomena are such as affect either directly or indirectly our temperature sense. By the temperature of a body is meant its condition as regards its ability to impart heat to or receive heat from other bodies. If two bodies possessing different temperatures be brought into thermal union, heat flows from the one of higher temperature to the one of lower temperature, and in general the flow of heat is such as to produce and maintain an equilibrium of temperature in the body, or system of bodies. When heat is applied to a body the following effects may be noted:

(a) The temperature of the body rises.

(b) The body undergoes a change in volume; in general an increase in volume of the body attends an increase in temperature.

(c) The body may change its state or condition, as for example, ice changes to water and water to steam upon the application of definite quantities of heat.

CALORIMETRY.

All measurements in heat must be made to depend upon some one of the above effects, usually that of change of volume. Temperature is defined quantitatively, by means of the expansion of a perfect gas, by assuming that equal increments of temperature produce equal increments of vol-

ume or pressure in the gas. As points of reference in the measurement of temperature the two fixed points for water, the freezing and the boiling points under standard conditions have been chosen. In the centigrade scale the temperature of melting ice is called 0°, and the temperature of steam forming freely under a pressure of 760 mm., of mercury, is taken as 100°. Temperatures are usually measured by means of mercury-in-glass thermometers. Such thermometers possess numerous disadvantages as compared with the gas or air thermometer, prominent among which are the following:

(a) Inequality of the bore of the glass tube.

(b) Inequality of the scale.

(c) Neither glass nor mercury expands equally and and uniformly throughout any large range of temperature.

(d) The instability and uncertainty of the fixed points of the thermometer, arising either from slow changes going on in the thermometer itself or from sudden and large variations in temperature incident upon the use of the thermometer.

From these causes it is evidently a matter of first importance to verify the readings of a mercury-in-glass thermometer, with which any accurate work is to be attempted.

Exercise 31. Determination of the Fixed Points of a Thermometer. The fixed points of an ordinary thermometer are usually in error by some fraction of a degree and these errors when determined, form the basis of correction to be applied to all subsequent readings made with the instrument. The fixed points must be frequently determined. These determinations fall under two heads

(*a*) *The Zero Point.*

The thermometer to be tested having been previously washed in water, is plunged into a clean vessel containing a mixture of distilled water and pure ice so that the mercury thread is entirely surrounded by the mixture. When the reading is taken the thermometer should be raised just high enough to show distinctly the upper end of the mercury filament. In reading care should be taken to avoid parallax. As soon as the mercury has fallen to 1°, note the reading every minute until it has remained stationary for five minutes, taking the final readings as the zero reading of the thermometer. Remove the thermometer from the ice mixture and allow the mercury to rise to 5°, after which repeat the experiment. Take the mean of the two results as the reading of the thermometer at 0°C. If this reading be a degrees, then the correction to be applied for the zero point is — a°.

(*b*) *The Boiling Point.*

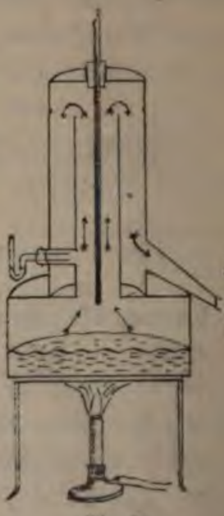
Fig. 29.

The apparatus (Fig. 29), consists of a brass vessel partly filled with water and having in its upper part double walls so arranged that the steam passes up through the inner cylinder, down through the outer space and escapes from a short tube near the bottom. The thermometer is passed snugly through a close-fitting cork, into the inner cylinder. The bulb should not come into contact with the water and, if possible, almost the entire filament of mercury should be enclosed by the steam issuing freely under at-

mospheric pressure. The temperature of the steam is found from the barometric pressure under which the water boils, from the following approximate formula:

$$t = 100^\circ + 0.0375\,(B - 760), \qquad (45)$$

where B denotes the barometric pressure in mm., corrected to 0°C. For accurate values of boiling point at different pressures see Table VI.

No readings should be taken until the steam issues freely from the tube at the bottom. This tube should be kept entirely open and the water should not be boiled too violently. (Why?) Why is there no correction for barometric pressure applied to the freezing point? What will be the correction for the boiling point if the thermometer reading is found to be $(t + b)^\circ$ C.? After the determination of the boiling point redetermine the freezing point. It will in general be found that the zero point has been "depressed" below the one first found, owing to the inability of the glass to follow immediately the sudden changes of temperature.

Under the supposition that the bore of the tube and the scale are uniform between the two fixed points, the value of a scale part may be found in terms of degrees by dividing t degrees by $(t + b - a)$. Calculate by this method the value of the tenth, twentieth, thirtieth, etc., divisions of the scale. Compare the values so found with a calibration obtained by direct comparison of the thermometer with a standard thermometer.

For this comparison both thermometers should be placed in a large bulk of water in such a way as to keep their bulbs near together, and the water constantly stirred during the comparison. Change the temperature of the

water by steps of 5° at a time from 0° up to 50°, and read both thermometers each time after the water has been well stirred. Compare the corrected table obtained by the comparison with the standard, with the calibration obtained by calculation.

Determine the fixed points of an ordinary thermometer and calibrate it from 0° to 50° C.

FORM OF RECORD.

Exercise 31. To determine the fixed points of a thermometer.

Thermometer No. .... **Date** ....

Zero point.		Boiling point.			Calibration.	
Reading	**Correction**	**Reading**	**t**	**Correction**	**Compared**	**Computed**
....	....	....	......	....	....	....
....	....	....	......	....	....	....

Depressed zero point. **Value of one scale part.**

SPECIFIC HEAT.

Temperature is to be sharply distinguished from quantity of heat. The former has reference only to the kinetic energy of the molecule and has no reference to the amount of matter involved. In *quantity* of heat account must be taken both of the temperature of the body and of its mass. The unit of temperature is the degree centigrade. The *unit of quantity of heat is the calorie.* A *calorie* is the quantity of heat required to raise the temperature of one gram of water 1° C. In this text no account will be taken of the change in the specific heat of water with varying temperature. The *thermal capacity* of a body is the number of calories required to raise the temperature of *the body* one degree centigrade. The *specific heat* of the substance, of which the body is composed, is its *thermal capacity per unit mass*, or it is the heat in calories required to raise the temperature of one gram of the substance one degree centigrade.

From the definition of a calorie it follows that *the specific heat of water is taken as unity.* The specific heat of any substance is different for different temperatures and hence as usually given, it denotes the mean value for the specific heat between certain temperature limits. The specific heat of water varies slightly with the temperature but may with but slight error be considered as constant.

METHOD OF MIXTURES.

If two substances of mass m_1 and m_2, at temperatures t_1 and t_2, and the specific heats s_1 and s_2, be brought into contact, they will come to some intermediate temperature t, such that the number of calories given out by the first is exactly equal to the number gained by the second, provided of course, that no heat has been lost externally through conduction or radiation. Then the equation for the heat exchange is

$$m_1 s_1 (t_1 - t) = m_2 s_2 (t - t_2) .$$

If the second substance be water, then $s_2 = 1$ and the equation is

$$s_1 = \frac{m_2 (t - t_2)}{m_1 (t_1 - t)} . \qquad (46)$$

In actual practice it is impossible to avoid loss of heat both by radiation and by conduction. If the water be contained in a vessel or calorimeter, then the latter receives heat along with the water and finally comes to the common temperature t. Let m be the water-equivalent or thermal capacity of the calorimeter; that is, let it take m calories to warm the calorimeter 1° C., then the total heat gained by the water and calorimeter will be $(m + m_2)(t - t_2)$ and we have

$$s_1 = \frac{(m + m_2)(t - t_2)}{m_1 (t_1 - t)} . \qquad (47)$$

Exercise 32. To Find the Water-Equivalent of a Calorimeter.

The calorimeter is a cup of thin metal, preferably of aluminium, which is placed inside a large vessel upon a flat piece of cork or other poor conductor. In the calorimeter is a stirrer and a thermometer. Let m be the water equivalent in calories of the calorimeter including stirrer and thermometer; also let the calorimeter contain m_2 grams of water at a temperature t_2; suppose the resulting temperature, due to adding m_1 grams of water at t_1, finally comes to be t. Then the exchange of heat is represented by the equation

$$(m_2 + m)\ (t_2 - t) = m_1\ (t - t_1)$$

or, solving for m

$$m = \frac{m_1\ (t - t_1)}{t_2 - t} - m_2\ . \qquad (48)$$

In practice weigh the calorimeter empty and dry, then fill about one-third full with water at a temperature about fifteen degrees above the temperature of the room and weigh again. The difference is m . Next add water of a temperature, about ten degrees below room temperature, until the resulting temperature after vigorous stirring is about room temperature. The temperature of the cold and warm water should be carefully determined, just before mixing, by means of a thermometer reading to tenths of a degree centigrade. The resulting temperature is to be taken only after the thermometer reading has become constant. Repeat the experiment twice, taking the mean of the three results as the water-equivalent.

For the correction due to radiation see later.

FORM OF RECORD.

Exercise 32. To determine water-equivalent of a calorimeter.

Date............

Weight of vessel	Vessel with water 1st.	m_2.	t_2.	t_1.	t	Vessel with water 2d.	m_1.	m.
......	......	..	..	..		......	..	..
......	......	..	..	..		......	..	..

Compare result with that obtained by multiplying mass of calorimeter by specific heat of the metal, as given in Table VII.

Exercise. 33. Specific Heat of Copper.

The piece of copper whose specific heat is to be determined is heated in a brass tube (Fig. 30), which is surrounded by a steam jacket. The copper is hung by a thread in the middle of the tube and the top is closed by means of a cork carrying a thermometer. The heater sits upon a wooden support having a hole of the diameter of the inner tube and coinciding with it. Upon this support slides a board provided with a similar hole and so arranged that the two holes coincide when the board is pushed in as far as possible. Immediately under the hole in the support is placed the calorimeter so that the heated body may be passed directly from the tube through the support into the calorimeter. The sliding

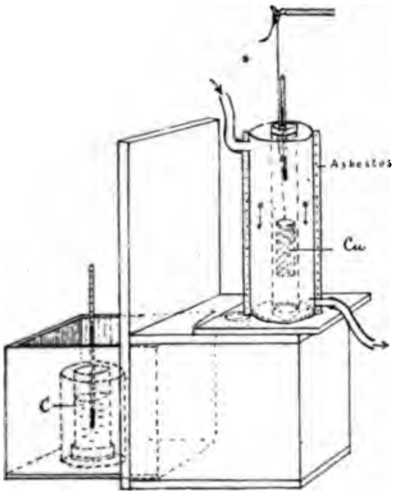

Fig. 30.

board is to shield the calorimeter from heat during the heating of the copper.

In use the dry copper is weighed and hung in place, the hole in the support closed by the sliding board and steam passed through the jacket until the temperature of the interior becomes constant, t_1. This heating usually requires about fifteen minutes. Meantime the calorimeter with the contained water is carefully weighed. The temperature of the water in the calorimeter is then read, t_2. The calorimeter is put in position, the sliding board pushed in, and the heated copper dropped gently into the vessel beneath. The calorimeter with its contents is then removed, the water thoroughly stirred and the highest temperature t, carefully noted. In order to avoid the necessity of correcting for radiation, it is well to have the temperature of the water in the calorimeter some 4° or 5° below the temperature of the room at the beginning of the experiment. Apply formula (47).

FORM OF RECORD.

Exercise 33. To determine the specific heat of copper.

Date................

Mass of calorimeter	Calorimeter with water.	m_1	m_2	m	t_1	t_2	t
......	......	....	....	....	....	....	....
......	......	....	....	....	....	....	....

Specific heat = ...

Exercise 34. Heat of Fusion of Water.

Water absorbs definite amounts of heat energy on passing from the solid to the liquid, and from the liquid to gaseous state. The quantities of heat thus absorbed are termed the heat of fusion and the heat of vaporization. The number of calories necessary to change one gram of ice at 0°C. to water at 0°C. is called *the heat of fusion of water.*

This may be measured in various ways. One of the simplest is by the method of mixtures; i. e., a known mass of ice at 0°, is added to a definite mass of water at a known temperature, and the temperature of the water at the end of the melting enables us to compute the amount of heat consumed in melting the ice.

Thus let m_2 grams of ice at zero, be added to m_1 grams of water at t_1, and let the temperature at the end of the melting be t; also let the water equivalent of the calorimeter, stirrer and thermometer be m, and let l denote the heat of fusion of ice as defined above. Then since the water formed by the melting of the ice must be warmed to t degrees, we have the heat lost by the calorime r and its contents equal to the heat absorbed by the ice and the ice-water. Hence

$$m_2 l + m_2 t = (m_1 + m)(t_1 - t)$$

from which

$$l = \frac{(m_1 + m)(t_1 - t)}{m_2} - t. \qquad (49)$$

The apparatus consists of a calorimeter, a thermometer, and a circular stirrer covered with wire gauze to keep the pieces of ice under water while melting. Weigh the calorimeter, fill nearly full of water at a temperature t_1, about fifteen degrees above room temperature and weigh again. The difference is the mass of water m_1. Break clean ice into small pieces and add to the water sufficient *dry* ice to bring the temperature of the calorimeter and its contents to about fifteen degrees below room temperature, when all the ice is melted. Stir vigorously throughout the operation, read the temperature t, as soon as the ice is all melted, and weigh the calorimeter and its contents once more. The difference between the last two weighings

gives the mass of the ice m_2, that was added.

FORM OF RECORD.

Exercise 34. To determine the heat of fusion of water.

Weighings of calorimeter: Date............

Alone	With m_1	With $m_1 + m_2$	m_1	m_2	m	t_1	t
......	......	......	....	....	....	....	....
......	......	......	....	....	....	....	....

Heat of fusion of water = ...

Exercise 35. Heat of Vaporization of Water at Boiling Point.

The heat of vaporization at the boiling point is the number of calories required to change one gram of water at that temperature into steam at the same temperature. Conversely if one gram of steam at this temperature be condensed into water the same number of calories will be liberated. Thus if m_2 grams of steam at a temperature t_2, having been conducted into a calorimeter of water equivalent m, containing m_1 grams of water at temperature t_1, produce by condensation and cooling, a resultant temperature t, we may write our equation of heat thus:

$$m_2L + m_2(t_2 - t) = (m_1 + m)(t - t_1)$$

or

$$L = \frac{(m_1 + m)(t - t_1)}{m_2} - (t_2 - t). \qquad (50)$$

where L is the heat of vaporization of water. The water equivalent m, of the calorimeter, may be found experimentally as before, or by multiplying the mass of the calorimeter by its specific heat.

The determination may be made by either of the following methods:

(a) Steam generated in a suitable flask is passed through a wide tube some 15 cms. long and 3 cms. wide,

(Fig. 31), in which the water from condensation is caught and retained. From this it passes directly into the water in the calorimeter. The mass of steam condensed m_2, is determined from the increase in weight in the calorimeter. The calorimeter having been carefully dried and weighed is filled nearly full of water at a temperature some fifteen or twenty degrees below that of the room and again weighed. The difference in weight is m_1. After the steam passes freely from the vertical tube leading from the water trap, the calorimeter and its contents are brought into place and the steam passed directly into the water until its temperature is some fifteen or twenty degrees above the temperature of the room. The water should be vigorously stirred during the condensation.

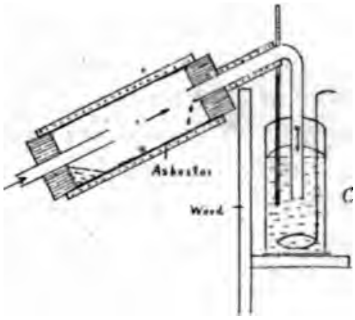

Fig. 31.

The calorimeter is then removed and the stirring continued until the temperature reaches a maximum, when the reading t, is taken and recorded. The temperature t_2, of the steam entering the calorimeter is to be determined by taking the barometric reading at the time of the experiment, and referring to Table VI. A third weighing determines the mass of steam condensed m_2.

Since m_2 is usually a small mass any loss of water due to drops adhering to the exit tube from the water trap leads to a relatively large error in the mass of steam condensed, and should be taken into account for accurate work. If steam be allowed to enter the calorimeter too rapidly, the tube leading into it is covered on its inner surface

with minute drops which are difficult to recover. Owing to the large influence of the above sources of error the following method is to be preferred.

(b) The apparatus (Fig. 32), consists of a closed copper vessel or calorimeter, provided with a stirrer and an opening for a thermometer; inside the closed vessel is a second smaller vessel into which steam is passed and there condensed. The calorimeter proper having been carefully dried and weighed, is filled nearly full of water at a temperature some 15° to 20° below room temperature and again weighed. The difference is m_1. Steam is then generated in a small glass retort connected with the inner vessel into which the steam passes and condensing gives up its heat to the calorimeter and its contents. The temperature of the cold water t_1, is to be taken just before the steam enters the calorimeter.

Fig. 32.

Steam is allowed to pass in until the resulting temperature rises as much above room temperature as the initial temperature of the water was below it. The flame is then removed and the temperature t, carefully determined. The amount of steam condensed is found by weighing the small retort before and after the experiment. The difference is the mass of steam condensed m_2. The temperature t_2, of the steam entering the calorimeter is to be determined from the barometric reading as before. Care must be taken to prevent the heat radiated by the

retort and the flame under it, from reaching the calorimeter. Stirring should be continued after removal of the flame until the temperature ceases to rise.

FORM OF RECORD.

Exercise 35. To determine the heat of vaporization of water.

Date..............

Weight of calorimeter	= ....
Specific heat of calorimeter	= ....
Water equivalent m	= ....
Weight of calorimeter with water	= ....
m_1	= ....
Weight of retort before experiment	= ....
Weight of retort after experiment	= ....
m_2	= ....

$m_1 + m$	t_1	m_2	t_2	t	L
......	....	....	....	....	....
......	....	....	....	....	....

CORRECTION FOR RADIATION.

In the preceding experiments the temperatures were so chosen as to render corrections for radiation and absorbtion unnecessary. In experiments requiring a greater degree of accuracy this loss or gain of heat by the calorimeter must be taken into account. This is best done by noting times and temperatures for an interval of at least five minutes before the instant at which the experiment proper begins, that is, the instant at which the ice, steam or metal enters the calorimeter. Readings should be taken every twenty seconds. Plot the times is abscissae and the temperatures as ordinates, (Fig. 33). After the beginning of the experi-

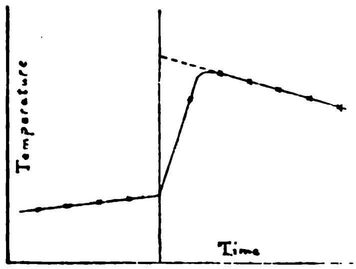

Fig. 33.

ment the temperature rapidly rises or falls, and having reached a maximum or a minimum it will practically become a linear function of the time for a few minutes, showing the rate of radiation or absorption. Produce this straight line backward to the axis. The point of intersection gives us with sufficient accuracy the temperature, which we should have obtained if there had been neither radiation nor absorption.

CHAPTER VII.

MEASUREMENTS IN SOUND.

Exercise 36. Velocity of Sound in Metals. (Kundt's Method.)

A brass rod held firmly clamped at its middle point when stroked with a resined cloth vibrates longitudinally like the air in an open organ pipe when sounding its fundamental tone. The middle of the rod being rigidly fixed is obviously a node, and the length of the rod is therefore *the half wave-length in brass of the sound produced.* If the end of the rod be brought into contact with an enclosed column of air whose length may be varied at will, it is possible so to adjust the length of the air column as to render it capable of vibrating in unison with the rod. In this case the enclosed air column having been thrown into stationary vibration behaves as a resonator closed at both ends; it must therefore contain at least one, and usually contains a number of *half wave-lengths* $\frac{1}{2}$, *of the sound in air*, produced by the rod. From the fundamental equation connecting velocity, frequency and wave-length, we have

$$n = \frac{V}{L} = \frac{v}{l} \text{ .*} \qquad (51)$$

where V, v, L, and l denote the velocities and wave-lengths of the same sound in brass and in air respectively. From this we have at once

$$V = v \cdot \frac{L}{l}$$

where v, the velocity of sound in air must be corrected for temperature according to the formula.

$$v_t = v_0 \sqrt{(1 + \alpha t)} \text{ †} \qquad (52)$$

where α for air at ordinary humidity, may be taken as 0.004.

In practice a brass rod about one centimeter in diameter and one meter long is clamped in a vise at its middle point and bears at one end a small disk of paper. A glass tube about 5 cms. in diameter and 150 cms. long (Fig. 34),

Fig. 34.

has one end closed air-tight by a sheet of rubber membrane tied smoothly over the end, while the other end is furnished with an adjustable piston sliding freely in the tube. The walls of the tube are lightly dusted throughout with fine cork filings or amorphous silica. The tube is placed horizontally upon two V shaped wooden supports so that the rubber membrane presses lightly against the disk of stiff paper on the end of the rod. The rod is set in vibration by chafing it gently with a piece of cloth or chamois skin covered with powdered resin. The cloth or chamois skin should be held between the thumb and fore

*Carhart, University Physics, Vol. I, p. 153.
†Kohlrausch, Physical Measurements, p. 138.

finger of each hand and pressed tightly against each side of the rod. When the rod is properly clamped a very slight pressure is sufficient to produce a loud clear tone. Adjust the piston in the outer end of the tube until the enclosed air vibrates freely on stroking the rod. This is indicated by the powder being tossed about in the tube and falling in the characteristic figures shown above. The nodes are indicated by small rings of powder and the antinodes by transverse layers or striae. The value of $\frac{1}{2}$ is found by measuring over a number of circles from center to center, and dividing the distance by the number of spaces or loops measured, remembering that the distance from node to node is equal to $\frac{1}{2}$. It will be noticed that a node is found both at the rubber diaphragm and at the piston. Why?

Measure over as large a number of loops as possible and compute $\frac{1}{2}$. Tap the tube lightly, rolling it over and over until the powder is evenly distributed, and repeat the determination. Take at least five separate sets of measurements. Avoid heating the rod by undue pressure or by continued rubbing.

FORM OF RECORD.

Exercise 36. To determine the velocity of sound in brass.

Temperature .... Length of rod.......... Date ....

Number of loops between nodes	Distance.	$\frac{1}{2}$		
....	....	....	$\frac{L}{2}$	....
....	....	....	$\frac{L}{1}$	....
....	....	....		
....	....	....	v	....
....	....	....		

Mean ....

Velocity of sound in brass ....

Exercise 37. Computation of Young's Modulus.

From the equation for the velocity of sound in any medium,

$$V = \sqrt{\frac{e}{d}},$$

where e is the coefficient of elasticity, and d the density of the medium in question, we may compute e at once from the data obtained in previous experiments. In the case of longitudinal waves transmitted through solids the coefficient of elasticity involved is M, Young's modulus,

whence $M = V^2 d$.

FORM OF RECORD.

Exercise 37. To compute Young's modulus for brass from velocity of sound and density.

Date ....

V as found in Exercise 36 ....
D " " " " 8 ....
M ....

Compare result with that obtained in Exercise 13.

Exercise 38. Rating a Tuning Fork. Graphical Method.

A tuning fork, one prong of which is armed with a fine sharp tip of flexible sheet copper, is mounted at right angles to a metallic cylinder. The cylinder is carried upon an axis furnished with a thread so that when rotated it is advanced longitudinally at the same time, so that the tracing point generates a spiral upon the surface of the cylinder. If the surface of the cylinder be slightly smoked, the tuning fork set in vibration and the cylinder rotated uniformly the tracing point describes a sinusoidal curve upon its surface. The number of vibrations executed in a second being thus automatically recorded by the fork, it is only necessary to indicate the beginning of the successive seconds upon the curve in order to read the vibration

frequency of the fork directly from the surface of the cylinder.

In practice the cylinder is covered with a sheet of firm smooth paper pasted smoothly on by gumming one end of the paper and pressing the other upon the gummed surface. *The paper must not be stuck upon the cylinder.* The paper is then smoked uniformly and lightly by means of a gas flame passed back and forth near the paper while the cylinder is continuously rotated, allowing only about an inch of the tip of the flame to touch the paper.

Care should be taken not to smoke the paper too black. The fork is then adjusted in its holder so that the point just touches the paper at the highest part of the cylinder and at the left end of the cylinder so that when the latter is rotated the fork seems to move from left to right along the cylinder. *The cylinder must rotate from the tracing point.* The time intervals are recorded upon the paper by connecting the cylinder and the fork to the secondary terminals of an induction coil, the primary circuit of which contains a suitable battery and is closed by a pendulum beating seconds. Consequently when the tracing point rests upon the paper and the coil is put in action a spark passes from the point to the cylinder each

Fig. 35.

time the circuit is closed, i. e., every second. The passage of this spark leaves a small spot on the smoked surface thus marking the time very accurately. The appearance of the sparks should be like that shown in Fig. 35.

Sometimes the coil will give two or three sparks instead of one. (Why?) In this case read *from the first one.* The fork should be so adjusted that when the point touches

the paper and the fork is properly bowed it will continue to vibrate for at least twelve seconds before coming to rest. When all the adjustments are made the coil is put in action, the fork bowed and the cylinder rotated uniformly while the fork continues to vibrate. The fork is then bowed again and the cylinder rotated as before. When the paper is filled the fork is removed, *the paper cut along the lap*, parallel to the axis of the cylinder almost but not quite apart; the cylinder is then turned over and the paper broken loose. In this way only can the paper be removed from the cylinder without spoiling the record.

The paper is then passed, face upwards, through a fixing solution of shellac in alcohol and then dried. In a few minutes the curve is ready to be examined. Count the waves for ten seconds and record. In counting the waves always take an even number of seconds.

FORM OF RECORD.

Exercise 38. To determine the frequency of a tuning fork by the graphical method.

Date . . . .

Seconds. : Number of Vibrations. : N

CHAPTER VIII.

ELECTRICAL MEASUREMENTS.

Units and Standards. *

(*a*) *Resistance.*

The practical unit of resistance is the ohm. It is represented by the resistance offered to an unvarying current by a column of mercury at the temperature of of melting ice, 14.4521 grams in mass, of a constant cross-sectional area and of length 106.3 centimeters.

For practical purposes standard ohms and multiples of the ohm are made of coils of wire, usually of some alloy whose resistance varies but little with the temperature, and which has a small thermo-electromotive force against copper, mounted in suitable protective cases. When these coils have been carefully adjusted by comparison with the original standard, they are issued by bureaus of weights and measures and serve as legal standards of resistance. See Figs. 36 and 37.


Fig. 36.

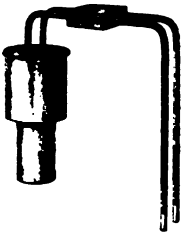
Fig. 37.

* With the exception of the volt, the units hereinafter described were adopted by the International Electrical Congress at Chicago, in 1893, and are frequently referred to as "international units." For the definition of the volt, see page 111.

For ordinary measurements, resistance boxes containing a number of coils of insulated wire, wound on bobbins non-inductively,* are in general use. The top of the box containing the coils is usually of ebonite and carries on its upper surface a number of heavy brass blocks so arranged that connection between adjacent blocks may be made by means of plugs inserted between them. The ends of the separate coils (Fig. 38), are fastened to the ends of adjacent blocks, so that when any plug is removed the current passes from one block to the next by passing through the connecting coil. In this way any resistance may be added by removing the proper plugs.

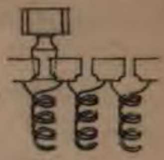

Fig. 38.

When the plug is inserted the current passes from block to block through the plug itself, the resistance of which must of course be negligible. On this account the plugs must fit accurately and be kept bright and clean, the ebonite must at all times be kept free from dust or moisture and *never be allowed to stand in the sun*, as the ebonite disintegrates slowly under the action of sunlight, a conducting layer of sulphur forms on the surface of the ebonite and the efficiency of the coils is impared. The plugs should at all times be handled by their hard rubber tops, they should be inserted with a gentle twisting motion, and *should all be loosened before the box is returned after use.*

Previous to the adoption of the ohm various other

* Two methods of non-inductive winding are used; in the one case the required length of wire is measured off, doubled upon itself and then wound, so doubled, upon the spool. In the second method the wire is wound in one direction only in each layer, but in opposite directions in consecutive layers. By this method the capacity of the coil, which by the first method may be quite appreciable, especially in coils of high resistance, is much reduced. (Chaperon).

units of resistance were employed and resistance boxes representing these earlier standards are frequently met in practice. The most common are the "British Association Unit," proposed by the British Association in 1864, and the "legal ohm" adopted at the Paris Congress in 1884. The relation between the ohm and these units is as follows:

1 ohm = 1.01358 B. A. units = 1.0028 legal ohms

1 B. A. unit = 0.9866 ohm.

1 legal ohm = 0.9972 ohm.

In reporting work in measurements of resistance the student *should state explicitly which units have been used.*

The term *rheostat*, (Fig. 39), is used for an unknown resistance of either fixed or variable value, and is usually employed in work with currents exceeding 0.1 ampere.

Fig. 39.

The ends of the lead wires connecting the different instruments should be bright and clean and clamped firmly by the binding posts, since loose connections offer *high* and *variable* resistance.

(*b*) *Current.*

The practical unit of current is the ampere. The ampere is represented sufficiently well for practical purposes by "the unvarying current which will deposit silver from silver nitrate at the rate of 0.001118 grams per second."

(*c*) *Electromotive force.*

The practical unit of electromotive force is the volt. The volt is the electromotive force which steadily applied to a conductor whose resistance is one ohm will produce a current of one ampere. The Clark cell is chiefly used as

a practical standard of electromotive force. This cell has for its positive electrode mercury covered by a paste of mercurous sulphate and a saturated solution of zinc sulphate, and for the negative electrode zinc amalgam, placed in the other leg of the H-shaped vessel, (Fig. 40), and covered by a saturated solution of zinc sulphate. Saturation of the zinc sulphate solution is secured by an excess of zinc sulphate crystals. The E. M. F. of this cell at the temperature t°C. is given by the formula

Fig. 40.

$$E_t = 1.433 - 0.0012\,(t - 15) \text{ volts } *$$

The legal form of this cell is not portable and suffers the additional disadvantage of a slight lag in the E. M. F., since the density of the solution adjusts itself to a new temperature but slowly, some time being required for the zinc sulphate crystals to dissolve or to crystallize out.

An excellent secondary standard is the Carhart-Clark cell, (Fig. 41), which is portable and avoids the temperature lag in the E. M. F. by saturating the zinc sulphate solution at 0°C. In this case the solution is not saturated at ordinary temperatures and hence no lag of E. M. F. behind the temperature occurs. Its E. M. F. is given by the formula

Fig. 41.

$$E_t = 1.440 - 0.00056\,(t - 15) \text{ volts.}$$

The temperature coefficient is thus one half that of the

* The value adopted by Congress for the E. M. F. of the Clark Cell is 1.434 international volts instead of 1.433 volts. Recent investigations (Carhart, Phys. Rev. Vol. 12, p. 129, 1901) show however that the value given above seems to be more nearly correct. The value used in Germany is $E_{15} = 1.4328$ volts.

Clark cell. *A Carhart-Clark cell should never be put on a closed circuit.* Why?

A new standard of E. M. F. has been proposed by Weston in which the zinc and zinc sulphate solution of the Clark cell are replaced by cadmium and cadmium sulphate. Its E. M. F. is given by the formula

$$E_t = 1.0186 - 0.00004\,(t - 20) \text{ volts.}$$

Standard cells are used in all cases where it is necessary to determine the absolute value of an E. M. F. or of a potential difference.

In many cases however, we wish simply to compare deflections of a galvanometer. In others the conditions may be so chosen that no current passes through the galvanometer. Such a method is termed a zero method. In experiments employing the zero method the Leclanche battery may be used. Where steady deflections are required however, a cell of constant electromotive force is necessary. In such cases we may use either a Daniell cell or a storage battery.

To set up a Daniell cell, (Fig. 42), first fill the porous cup containing the amalgamated zinc, or negative electrode, two-thirds full of zinc sulphate solution and wait until the solution begins to moisten the outside of the cup, before placing the cup in the glass jar containing the copper, or positive electrode, and the copper sulphate solution. In order to avoid changes in the internal resistance during the experiment it is well to short circuit the cell for ten minutes before using.

Fig. 42.

After use the cell must be taken apart, the zinc sulphate poured back into its bottle, the porous cup thoroughly washed and *the zinc rubbed clean.* Copper oxide is usually deposited on the zinc as a black film. This may be readily rubbed off while it is moist, but if it be allowed to dry it adheres firmly and renders it difficult to amalgamate the zinc again. The E. M. F. of a Daniell cell is about 1.1 volts and this value may be used in all cases where great accuracy is not required.

When a constant E. M. F. of more than one volt is required, a *storage battery* (E. = 2.2 volts) may be employed. On account of the very low internal resistance of a storage battery great care must be exercised to avoid short circuiting the cell. The student using a storage battery should *always leave one of the electrodes disconnected* until the instructor has seen and approved of the arangement of the apparatus.

(*d*) *Quantity of electricity.*

The practical unit of quantity is the coulomb. The coulomb is the quantity of electricity transferred by a current of one ampere in one second.

(*e*) *Capacity.*

The practical unit of capacity is the farad. The farad is the capacity of a condenser which is charged to a potential of one volt by a quantity of one coulomb. The microfarad 10^{-6} farad, is commonly used as the measure of capacity. Secondary standards of capacity are made in the form of condensers with solid dielectrics. A large number of sheets of tin foil interleaved with mica or paraffin are placed in a bath of melted paraffin, in a vacuum chamber to remove air bubbles, and allowed to

wool. Alternate sheets of tin foil are then connected and joined to one binding post, the remainder to another, thus forming the two ends or terminals of the condenser. A subdivided condenser, is made by connecting all the leaves of one set to one bar, Earth, (Fig. 43), as a terminal, and dividing the leaves of the other set into some number of divisions each of which is connected to a separate bar which in turn may be connected to a second binding post by means of a plug. When any division is to be used the plug is inserted for that division.


Fig. 43.

(f) Selfinductance.

The practical unit of selfinductance is the henry. A henry is the selfinductance in the circuit when the E. M. F. induced in the circuit is one volt, while the inducing current varies at the rate of one ampere per second.


Fig. 44.

The usual form of self-inductance (Fig. 44), consists of two coils in series, one fixed and the other movable about a diameter of the fixed coil as an axis. The movable coil may be rotated through 180° and in this way the selfinductance may be varied considerably. The instrument is calibrated empirically. The coils are wound on wood, and metal is, so far as possible, entirely avoided in the construction.

Instruments.

KEYS.

In most forms of apparatus for making electrical measurements, it is necessary to allow the current to pass through the measuring instrument for but a short space of time. Keys are provided for the purpose of closing and opening the circuit as may be desired. The ordinary key is so arranged that when pressed down, contact is made between two platinum points and the circuit is closed. On releasing the key the circuit is opened. For closed circuit work, plug keys or knife switches of various forms are used.

In work with the Wheatstone's bridge it is necessary to close the circuit through the galvanometer, *after* the current in the arms of the bridge has reached a constant value. For this purpose a double key is provided, in which the contacts for the circuit through the bridge arms, and that through the galvanometer are made *successively* in in the order mentioned. Such a key is termed a *successive contact key*.

It is frequently necessary to reverse the direction of the current through an instrument without loss of time. This is most conveniently effected by means of the Pohl's commutator (Fig. 45). This consists of four cups containing mercury connected by cross wires as indicated in the figure, and a light frame of wires by

Fig. 45.

which two other cups at the end of the block may be put in connection with the pair of cups on either side. If the source of current be connected to the pair of binding posts at the ends of the block, and the galvanometer to the two

binding posts on either side, the current is passed through the instrument in one direction or the other by tilting the frame from side to side.

All mercury contacts should be kept clean and should have the ends of the metal dipping into the mercury well amalgamated.

GALVANOMETERS.

The space about a magnet or about a wire carrying a current of electricity is called a *magnetic field*. Such a field is conceived to be filled with lines of magnetic stress or lines of magnetic force. The direction of these lines is assumed to be the direction along which a free north seeking pole would tend to move. The lines of magnetic force are said to run out from the north pole of a magnet, curve round through the air and reenter the magnet at the south pole. In the case of a wire carrying an electrical current the lines of force are concentric circles surrounding the wire. At any point distant r from a straight conductor in which a current is flowing, the strength of the magnetic field due to the current is directly proportional to the current and inversely proportional to the perpendicular distance between the conductor and the point.

Whenever two magnetic fields are brought near to each other there arises a stress between them, tending to turn the fields into such a position that they will mutually include the greatest number of lines of force. Obviously the moment of this stress will be greatest when the two systems of lines of force stand at right angles to each other. This is the position adopted in galvanometers and electro-dynamometers. In instruments designed to measure currents, at least one of the magnetic fields must arise

from the current flowing through a coil of wire and hence must vary as the strength of the current. The other magnetic field may be produced by a permanent magnet as in the galvanometer, or by a second coil carrying a current, as in the electro-dynamometer. One of the magnetic fields must be capable of rotation. Galvanometers may be devided into two classes: *

(*a*) *Galvanometers with stationary coil and movable system of magnets:* needle type; (Fig. 46).


Fig. 46.

(*b*) *Galvanometers with stationary magnets and movable coil:* D'Arsonval type. (Figs. 47 and 48).


Fig. 47.

In all measuring instruments the deflecting moment due to the current to be measured, must be balanced against a restoring or directing moment which tends to restore the system to its original position. When the system thus subjected to the action of two moments, comes to rest we know that the moments of the deflecting and restoring forces are equal and opposite in direction. The angle of deflection is determined in one of several ways and the current determined as a function of this angle. The directing force may be due to the action of an independent magnetic field upon the movable magnetic needle, or to the torsional moment of the suspending wire.

The sensitiveness of a galvanometer may be increased

* Carhart, University Physics, Vol. II, pp. 335–340.

by decreasing the directing or restoring force. In the *astatic galvanometer* the moving system is composed of two magnets or systems of magnets of nearly equal strength placed one above the other with poles opposed. One of the needles is placed within a coil, the other needle either outside, or better still enclosed in a second coil through which the current flows in the opposite direction. In this way the magnetic moment of the system with respect to the earth's field is greatly reduced.

On the other hand a magnet placed under or over a movable magnetic needle may be made to exert any desired directive force. Such a magnet is termed a controlling magnet. Some very sensitive galvanometers employ both an astatic system and a controlling magnet.

METHODS OF OBSERVATION.

In some instruments the moving system is furnished with a light pointer playing over a scale, but in the more sensitive galvanometers the deflections are observed by means of a mirror attached to the moving system. This mirror may be either concave or plane. In the first case an illuminated slit is focussed by the mirror upon a semi-transparent scale and the deflections are read directly from the scale. In the second case a telescope is employed to view the image of a scale reflected in a plane mirror. Both methods are in common use.

The D'Arsonval galvanometer, (Figs. 47—48), is more convenient for ordinary measurements. The damping effect is very large and the coil may be brought to rest almost at once by short circuiting the galvanometer. Why is this? The instrument is also practically independent of the surrounding magnetic field and is consequently free

from disturbances arising from variation in the intensity of this field, which often prove very troublesome in galvanometers of the needle type. The D'Arsonval galvanometer has the additional advantage that it may be placed in any position, while instruments of the needle type must be set so as to have the plane of the coils in the magnetic meridian. The needle type however has usually greater sensitiveness, although the D'Arsonval type is sufficiently sensitive for most purposes.

Fig. 48.

SHUNTS.

It frequently happens that the current to be measured will produce a deflection too great to be observed, or in some cases it may even endanger the instrument itself. In such cases we may reduce the current flowing through the galvanometer by means of a resistance connected in parallel with it. This resistance is called a shut. Let g be the resistance of the galvanometer, s that of the shunt, then the resistance of the two circuits in parallel is $\frac{g\,s}{g+s}$, and by Ohm's law, if I denote the total current and I_g the current through the galvanometer, we have

$$I_g : I :: \frac{g\,s}{g+s} : g$$

or

$$I_g = I\,\frac{s}{g+s}\,. \qquad (53)$$

If we observe the current I_g in the galvanometer, then

the total current is $I_g \cdot \frac{g+s}{s}$; where the factor $\frac{g+s}{s}$ is called the *multiplying power of the shunt.* Let the galvanometer resistance be n times that of the shunt, ($g = ns$), then the multiplying power is $\frac{n+1}{1}$. If n be 9, 99 or 999, the corresponding values of the multiplying power are 10, 100, and 1000. The makers frequently furnish with the galvanometer shunt-boxes containing resistances equal to 1/9, 1/99, and 1/999 of that of the galvanometer, in which case the observed current I_g, is equal to 0.1, 0.01 or 0.001 I.

Exercise 39. To Calibrate a Galvanometer by means of Ohm's law.

The object of this experiment is to determine whether the deflections of a galvanometer are proportional to the current flowing through it, or if that is not the case, to ascertain how the deflections vary with the current. To obtain currents which stand in a definite relation to each other we apply Ohm's law, by connecting the terminals of the galvanometer to different points of a circuit through a which a *constant current* is flowing. Then the potential difference, P. D., between the extremities of a resistance r, through which a current i is flowing, is equal to ir, and this potential difference causes the current through the galvanometer producing the observed deflection.

The simplest arrangement is to use a battery of constant E. M. F.* and to send the current through a straight

---

* The student may observe as a general rule, that a battery of constant E. M. F. is always to be used where CONSTANT DEFLECTIONS are to be obtained while ordinary cells may be employed for zero methods or ballistic methods.

wire, as for example, the wire of a slide wire bridge, (Fig. 49), and connect the galvanometer to two points, P and A, on this wire, where A denotes the position of the contact maker. Since the current through the wire must remain constant, no matter where the galvanometer is attached, it is evident that the galvanometer should have a very high resistance as compared with that of the wire. Explain this. The resistance of the wire of a slide wire bridge is usually about 0.2 of an ohm. If the galvanometer resistance is less than 2000 ohms, a resistance box R_2, with sufficient resistance to bring the resistance of the galvanometer up to 2000 ohms, should be put in series with it. The resistance box R_1 is inserted in the battery circuit in order to reduce the potential difference between P and A to a value such that with the maximum length of wire used in the experiment the deflections of the galvanometer will still be on the scale.

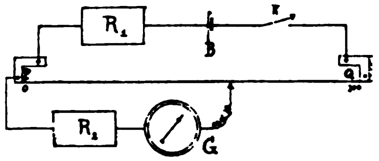

Fig. 49.

Move the point A, an ordinary contact maker, along the wire by steps of 5 cms., from 5 to 95 cms. on the scale and observe the successive deflections of the galvanometer. Next reverse the battery current and repeat the observations. The mean of the deflections and the corresponding lengths between P and A are plotted. The resulting curve will be a straight line, if the deflections of the galvanometer are proportional to the current flowing through it. What principle besides Ohm's law has been applied in this experiment?

FORM OF RECORD.

Exercise 39. To calibrate a galvanometer by means of Ohm's law.

Galvanometer No. .... **Date** ....
Resistance boxes **R_1** .... **R_2** .... Room No. ...

Length PA	**Deflection a.**	**Deflection b.**	**Mean Deflection.**
....	....	....	....
....	....	....	.

Exercise 40. To Determine the Figure of Merit of a Galvanometer.

The figure of merit f, of a galvanometer is that current which will produce a deflection of one scale division. In case the scale is movable the distance of the scale from the mirror must be specified. This exercise furnishes a simple application of Ohm's law.

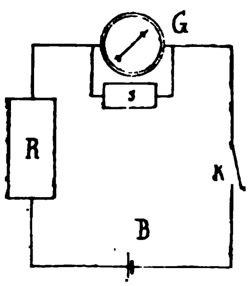

Fig. 50

The exercise may be performed in either of the following ways:

(a) The arrangement is as shown in Fig. 50. Let B denote a battery of constant E. M. F.; R a very high resistance; g and b the resistances of the galvanometer and battery respectively; then

$$I = \frac{E}{R + g + b}, \qquad (54)$$

if no shunt is needed, and

$$I = \frac{E}{R + \frac{g\,s}{g + s} + b}, \qquad (55)$$

if a shunt is used.

If the galvanometer show a deflection d, on the passage of a current I_g, through it, then we may assume, in the great majority of cases, that this deflection is proportional

to the current, and we have the relation

$$f\,d = I_g. \tag{56}$$

Hence in the first case

$$f = \frac{I_g}{d} = \frac{I}{d} = \frac{E}{(R+g+b)\,d}, \tag{57}$$

in the second case

$$f = \frac{I_g}{d} = \frac{I}{d}\cdot\frac{s}{g+s} = \frac{E}{\left(R+\frac{gs}{g+s}+b\right)d}\cdot\frac{s}{g+s}. \tag{58}$$

Usually b as well as g, and still more $\frac{gs}{g+s}$, are negligible in comparison with R, and the formulæ become much simpler.

(b) For sensitive galvanometers which have no permanent shunt the following method is convenient: Let B, (Fig. 51), be a cell of constant electromotive force E, and close the circuit through P and Q. Then take the potential difference over Q to produce the deflection of the galvanometer. Q must be very small in comparison with R. Now the applied potential difference is $\frac{Q}{P+Q}E$ and our formula for f becomes

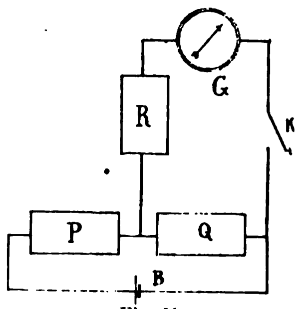

Fig. 51.

$$f = \frac{Q}{P+Q}\cdot\frac{E}{R+g}\cdot\frac{1}{d} \tag{59}$$

In practice vary the resistance R between 150000 and 250000 ohms and observe the deflections; in case (b) the ratio $\frac{Q}{P+Q}$, may also be varied.

The term sensitiveness of a galvanometer is frequently used in a different sense. It may be defined as

the resistance which the circuit must have in order that one volt may produce unit deflection.

$$s = \frac{1}{f} \cdot \qquad (60)$$

It should be kept in mind, that for a given potential difference at the terminals of the galvanometer, the deflection will not depend upon the sensitiveness alone, but upon the *resistance* of the galvanometer as well. The deflection is in this case inversely proportional to f r.

FORM OF RECORD.

Exercise 40. To determine the figure of merit of a galvanometer.

Galvanometer ....   **Date** ....
Distance of mirror from scale ....   **Room** ....

E	R	S	$\frac{Q}{P-Q}$	d	f
....	....	....	....	....	....
....	....	....	....	....	....

BALLISTIC GALVANOMETERS.

While in ordinary galvanometers it is required to observe deflections due to a steady current flowing through the instrument, or to prove the absence of a current from the absence of a deflection, it is often necessary to measure the quantity of electricity passing through the galvanometer, as in measuring the quantity of electricity stored in a condenser.

When a condenser is discharged through a galvanometer the current rises rapidly to a maximum, and then decreases to zero. In such a case a galvanometer having a coil with a large moment of inertia must be employed. Such an instrument is termed a ballistic galvanometer and its advantage consists in this, that the coil remains practically at rest until the entire quantity of electricity has passed through it. In this way the full force of the mag-

netic thrust is effective in starting the coil which moves off as if started by a blow. For small angular deflections the quantity of electricity may be set proportional to the deflections.* Here we observe the maximum deflection attained by the system on the first throw of the needle, and not a constant deflection. That quantity of electricity which gives unit deflection is called *the constant of the ballistic galvanometer*, or if the quantity Q give a deflection d, then

$$c = \frac{Q}{d}\,. \tag{61}$$

Exercise 41. To Determine the Constant of a Ballistic Galvanometer.

To determine the quantity Q giving a certain deflection d, use a battery of known electro-motive force E, as a standard cell, and charge a condenser of known capacity by depressing the key to the point b, (Fig. 52). Then by releasing the key, the condenser is disconnected from the battery and discharged through the galvanometer. A special key, used for such experiments, known as a charge and discharge key, is shown in Fig. 53.

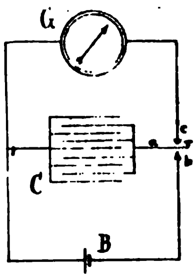

Fig. 52.

Letting c represent the constant of the ballistic galvanometer, Q the quantity discharged, and d the deflection, then

$$c = \frac{Q}{d} = \frac{EC}{d} \tag{62}$$

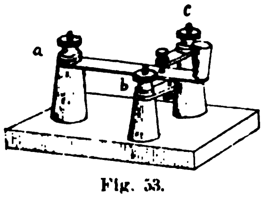

Fig. 53.

where C is the capacity of the condenser and E the voltage of the standard cell. In practice vary the quantity and take the mean of three different trials.

*Carhart and Patterson, Electrical Measurements, pp. 207-213.

FORM OF RECORD.

Exercise 41. To determine the constant of a ballistic galvanometer.

Date..........................

		E	C	Q	d	c
Galvanometer	Room	......	......	......	......	......
Condenser	Temperature	......	......	......	......	......
Standard Cell.........		......	......	......	......	......
					Mean =	......

VOLTMETERS AND AMMETERS.

Voltmeters and ammeters are usually portable galvanometers of the D'Arsonval type. Wall instruments designed to measure high voltages or large currents, as in electric lighting or power stations, are usually attached to the switch board directly and give continuous indication as to the pressure and volume of the current furnished. Voltmeters and ammeters are direct-reading instruments, that is, they are so calibrated as to show directly upon an arbitrary scale the difference of potential existing, or the current flowing between any two points to which they may be connected.

Fig. 54.

The best known instruments of this class are those designed by Weston. In these the directing force is furnished by two spiral springs of phosphor-bronze. Rapid damping is secured by the use of aluminium frames upon which the coils are wound. The voltmeter (Fig. 55) is commonly provided with two scales, one for high and one for low voltages. In the figure the low voltage terminal on the negative side is marked 15.

Fig. 55.

The Weston instruments are durable and remarkably accurate and the student should thoroughly familiarize himself with them before attempting to use them independently.

Measurement of Resistance.

Exercise 42. To Measure a Resistance by Substitution.

The apparatus is arranged as shown in Fig. 56. B is a battery of constant E. M. F., x the unknown resistance, R a resistance box, G a galvanometer, K a key which may connect the galvanometer either to x or to R.

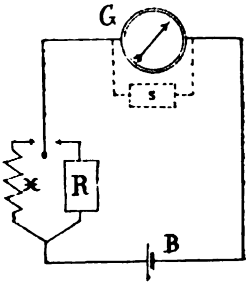

Fig. 56.

The battery circuit is first closed through the resistance x, and the galvanometer. If the deflection be too great, the controlling magnet or the torsion head may be so turned as to bring the deflection back upon the scale, or a shunt may be applied to the galvanometer. *Do not use a high resistance in the circuit instead of a shunt.*

Assuming that the deflections are proportional to the current we have

$$d_1 = c\,i_1 = c\,\frac{E}{x + r}, \qquad (63)$$

where r is the resistance of the whole circuit except x, and c is the proportionality factor. Next send the current through R instead of x; then

$$d_2 = c\,i_2 = c\,\frac{E}{R + r}. \qquad (64)$$

If now R be adjusted until the deflections in the two cases are equal, then

$$x = R.$$

If R can not be adjusted so a to produce exactly the

same deflection as x, interpolate between the two nearest values above and below. Repeat the readings at least three times.

FORM OF RECORD.

Exercise 42. To measure a resistance by substitution.

Galvanometer....	Temperature....	Date..............
Resistance of.....	Room...........	Resistance box....
Deflection with x	Deflection with R_1	Deflection with R_2
....	....	....

x = ... ohms.

Exercise 43. Resistance by Voltmeter and Ammeter.

This exercise consists in measuring *at the same time*, the current flowing through a wire and the difference of potential at its terminal points. Then if x be the resistance of the wire, we have by Ohm's law

$$X = \frac{V}{I}. \qquad (65)$$

The potential difference at the terminals is measured by means of a voltmeter whose resistance must be very high in comparison with the resistance to be measured, as otherwise the current through the resistance x, will not be the total current I, measured by the ammeter. This is easily seen by applying the shunt rule,

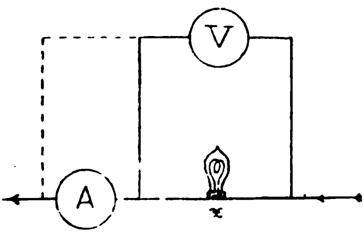

Fig. 57.

$$I_x = I\frac{R_v}{R_v + x}$$, where R_v is the resistance of the voltmeter. If the resistance x is too large, it is better to include the ammeter, whose resistance is very small, together with the resistance x between the terminals of the voltmeter, as shown by the dotted lines in

Fig. 57, and then subtract the resistance of the ammeter from the resulting value of x.

Measure the resistance of an incandescent lamp. Put a rheostat in series with the lamp and the ammeter. Cut out resistance from the rheostat step by step until the lamp has reached its full candle power. Observe at each step the readings of the voltmeter and ammeter. Calculate the resistance for each current, and the watts absorbed by the lamp.

FORM OF RECORD.

Exercise 43. To measure the resistance of an incandescent lamp by voltmeter and ammeter.

Date ....
Voltmeter No .... Milliammeter ....
Name of lamp .... Candle power ....

V	I	R	Watts
....	....	....	....
....	....	....	....

Exercise 44. Measurement of Very High Resistances by Direct Defletion.

This method is a modification of the last. Instead of an ammeter, a galvanometer is used whose figure of merit is determined as in Exercise 40. The formula for the resistance then becomes

$$x = \frac{V}{f\,d} - g, \qquad (66)$$

since $I = f\,d$.

Usually g, the resistance of the galvanometer, is negligible in comparison with the high resistance x. High resistances of this kind are insulation resistances, as for example, the resistance offered to the passage of a current by the insulation of a cable or of a condenser. It is advisable to insert a high resistance H. R. in series with the galvanometer, in order to protect the instrument from excessive currents in

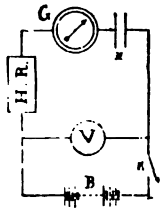

Fig. 58.

case of a break in the insulation. If the deflections vary notably with the time, it is advisable to keep the key K, closed and take a series of readings at definite time intervals. *If x represent a capacity short circuit the galvanometer by means of a shunt key, before closing the key K,* and afterwards open the short circuiting key to observe the steady deflection.

FORM OF RECORD.

Exercise 44. To measure the resistance offered by the insulation of a commercial condenser.

Condenser		Date		
Galvanometer	Time	V	d	x
Room	.....	.. .		
Figure of merit (40)	....	......	.....	...
Resistance of the galvanometer ...	......	....	.. .	....

THE WHEATSTONE BRIDGE.

The most accurate method for measuring resistance is by means of the Wheatstone bridge. This apparatus consists of a network of six conductors joining four points, A, B, C, D, (Fig. 59), so arranged that each point is joined to each of the other three points by separate conductors. Let one of the conductors contain a source of E. M. F.;

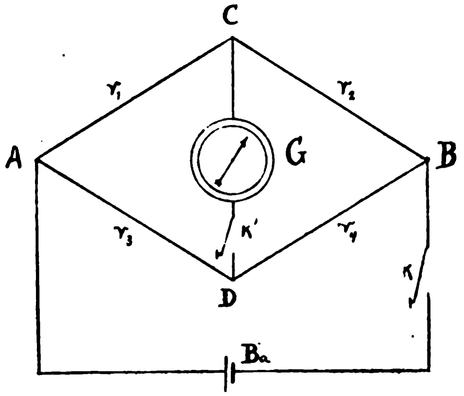

Fig. 59.

four of the others will form a divided circuit while the remaining one, containing a galvanometer, will form a bridge between the two parallel conductors. Let r_1, r_2, r_3, r_4 be

the resistances forming the four branches of the divided circuit, and suppose them to be so adjusted that no current flows through the galvanometer. Then it may be shown that the resistances satisfy the relation

$$\frac{r_1}{r_2} = \frac{r_3}{r_4}.$$

For let the potentials of the four points be represented by V_a, V_b, V_c, V_d, then since there is the same fall of potential between A and B, whether we pass by one route or the other, and since the fall of potential is at all times proportional to the resistance passed over, we have

$$\frac{V_a - V_c}{V_a - V_b} = \frac{r_1}{r_1 + r_2} \text{ and } \frac{V_a - V_d}{V_a - V_b} = \frac{r_3}{r_3 + r_4}. \quad (67)$$

But since no current passes through the galvanometer $V_a - V_c = V_a - V_d$ therefore

$$\frac{r_1}{r_1 + r_2} = \frac{r_3}{r_3 + r_4}, \quad (68)$$

whence

$$\frac{r_1}{r_2} = \frac{r_3}{r_4}. \quad (69)$$

From the above relation it is evident that if three of the resistances be known the fourth may at once be determined. In fact it is necessary to know but one resistance and the ratio between the other two. Since the current through the parallel branches should become steady before the potentials at C and D are tested, it is necessary to close the *galvanometer key last in every case*. A successive contact key is best adapted to this work. What would be the effect of self-inductance in one of the branches?

It will be found advantageous to follow Maxwell's

rule:* "Of the two resistances—that of the battery and that of the galvanometer—connect the greater resistance so as to join the two greatest to the two least of the four other resistances." This insures the greatest sensitiveness of the apparatus.

Since by this method the resistances are to be adjusted until no current flows through the galvanometer, we may use any source of E. M. F. whether constant or not. Such a method is called a zero method. The temperature of the room must be carefully noted.

Exercise 45. To Measure an Unknown Resistance by Means of the Post Office Box.

In the Post Office box, (Fig. 60), there are three known resistances connected in series, one of which may be given any value between one ohm and the maximum, usually 10,000 ohms. The other two resistances form the proportional arms of the Wheatstone bridge and contain but a few coils, usually 1, 10, 100 and 1000 ohms. In this way the ratio between the known and the unknown resistances may be varied from 1000 to 0.001. The galvanometer is generally connected to the outer ends of the proportional coils and the battery to the remaining two binding posts. Frequently the two keys and a galvanometer are included in the case thus making it a very compact and convenient instrument for the measurement of resistance.

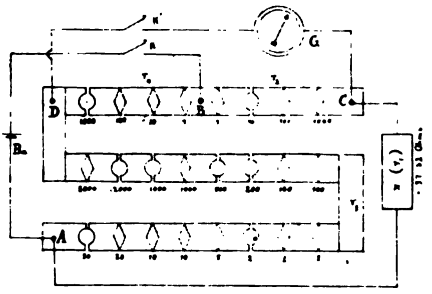

Fig. 60.

*Maxwell. Electricity and Magnetism. 3d edition. Vol. I. p. 478.

In practice, especially where the unknown resistance is not even approximately known, it is well to begin with equal resistances in the proportional arms r_2 and r_4. If the unknown resistance r_1, is too large the deflection on closing the galvanometer will be in one direction, if too small it will be in the opposite direction. First find two values for r_3 which change the direction of the deflection of the galvanometer, and *keep in mind the meaning of the direction of the deflection.* After having thus found the approximate value of the unknown resistance, change the ratio of the proportional arms so as to obtain the smallest value of the ratio $\frac{r_2}{r_4}$, that r_3 will allow and still produce a balance, and then make the final determination. Sometimes it may be necessary to interpolate between the values of r_3. Measure the resistance of three pieces of wire, and calculate the specific resistance of each metal from the formula $\rho = R\frac{a}{l}$, where R is the resistance, l the length and a the cross-sectional area of the wire.

FORM OF RECORD.

Exercise 45. To determine the specific resistance of three metals.

Postoffice box No......		Date	......
Galvanometer	......	Room	......
Temp. of room	......	Temp. of Coil......	

Specimen of wire	r_2	r_4	r_3	r_1	l	diam.	a	ρ
......	....	....	....	....	....	....	....	....
......	....	....	....	....	....	....	....	....

Exercise 46. Resistance by Slide-Wire Bridge.

From equation (69), it is seen that only the ratio $\frac{r_2}{r_4}$, and one resistance r_3, need be known, to effect the

measurement of an unknown resistance. The ratio may be furnished by the two parts of a wire of uniform cross-section. In the slide-wire bridge the sum of the lengths of the two wires representing r_3 and r_4 is kept constant, usually 1000 mms., and the ratio of their lengths is changed by moving one of the galvanometer terminals along the wire. For this purpose a contact maker k', (Fig. 61), is substituted for the galvanometer key in the Wheatstone bridge. If on closing K and making contact at k', no current flows through the galvanometer, we have, neglecting the very low resistance of the copper straps,

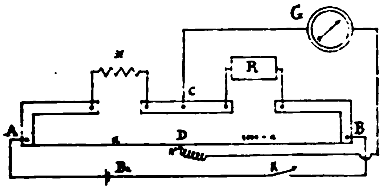

Fig. 61.

$$x = R \cdot \frac{l_3}{l_4} \qquad (70)$$

or letting the reading at B on the wire equal a mms., then

$$\frac{x}{R} = \frac{a_1}{1000 - a_1}. \qquad (71)$$

To obtain accurate results we must interchange x and R, in order to correct for possible errors due to variations in the cross-section of the wire, or to unsymmetrical placing of the scale or to a constant error in the reading of the position of the contact maker. After exchanging x and R, a new reading, a_2, is obtained; then

$$\frac{x}{R} = \frac{1000 - a_2}{a_2}, \qquad (72)$$

or combining (71) and (72)

$$\frac{x}{R} = \frac{1000 + (a_1 - a_2)}{1000 - (a_1 - a_2)}. \qquad (73)$$

As shown on page 9, the errors of observation influence the result least when the contact maker is at the center of the wire. That is when x and R are equal.

In some bridges four gaps are provided in the large copper strips for the insertion of resistances. In such bridges the two inner gaps are for the resistances x and R, while the outer ones may be used to insert two nearly equal resistance coils which serve as an extension of the slide wire, thus enabling the experimenter to determine the ratio of the two lengths with greater accuracy. These additional coils must be determined in terms of mms. of the bridge wire.*

Avoid moving the contact maker K' while pressed down. A slight pressure should suffice to make contact. If not the key and wire should be cleaned.

FORM OF RECORD.

Exercise 46. To measure the resistance of an electromagnet. Study effect of self-inductance by closing K' before K.

Slide-wire bridge No. . . . . Date . . . .
Galvanometer . . . . . . Temperature . . . . . . Room . . . . . .

R	a_1	a_2	x
. . . .	. . . .	. . . .	. . . .
. . . .	. . . .	. . . .	. . . .

Exercise 47. Resistance of a galvanometer. (Thomson's Method).

The principle of the Wheatstone bridge may be applied to the measurement of the resistance of a galvanometer. Here the galvanometer is put in one of the four arms of the bridge and the battery circuit closed, producing a deflection of the galvanometer. If the deflection is too great a resistance may be introduced into the battery cir-

* Carhart and Patterson. pp. 58-64.

cuit, to reduce the current—or the controlling magnet may be lowered or the torsion head twisted until the deflection comes once more upon the scale. Instead of a galvanometer as in the ordinary arrangement, a key is inserted, shown as k′ in Fig. 62, or the contact maker may be used as before. If with a steady deflection of the galvanometer this key be closed and a current pass between C and D in either direction then the

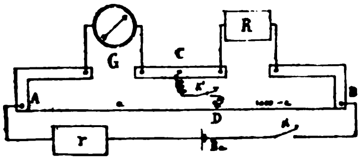

Fig. 62.

deflection of the galvanometer will change. Only when there is no current through k′, will the scale reading remain constant. When a constant scale reading with k′ open or closed is attained, the condition for a balance in the bridge is fulfilled. Then

$$g = R \frac{a}{1000 - a} \text{ ohms.} \tag{74}$$

In this case also the best results are obtained by making R = g. Exchange g and R and apply formula (73).

At first any resistance R_1, may be taken and the balance sought, simply to find g approximately. Then make R as nearly equal to g as possible and repeat with greater care. Must a cell of constant E. M. F. be used in this experiment?

FORM OF RECORD.

Exercise 47. To measure the resistance of a galvanometer.

Date. . . . .

Room. . . . Temperature. . . . Galvanometer No. . . . .

Preliminary.		Final.			
		R	a_1	a_2	g
R_1 =	. . . .	. . . .	. . . .	. . . .	. . . .
a =	. . . .	. . . .	. . . .	. . . .	. . . .
g =	. . . .	. . . .	. . . .	. . . .	. . . .

Exercise 48. Resistance of a Galvanometer. (Second Method.)

The resistance of a galvanometer may also be determined by the application of the principle involved in Exercise 39. The apparatus is arranged as shown in Fig. 49. With no resistance in R_2 the deflection of the galvanometer is observed for a length of bridge wire a_1, of about 30 cms. The length of the wire is next made about twice as large, that is 60 cms. approximately, and sufficient resistance is added in R_2 to make the deflection of the galvanometer approximately equal to the first deflection. Finally move the contact key along the wire until the deflections are exactly equal. Call this reading of the bridge wire a_2. Then neglecting the resistance of the wire in comparison with that of the galvanometer

$$g = \frac{a_1}{a_2 - a_1} \cdot R_2$$

FORM OF RECORD.

Exercise 48. To measure resistance of a galvanometer by second method.

Date

Galvanometer No....... Temperature...... Room......

a_1	a_2	R_2

$g = \ldots\ldots\ldots\ldots$

Exercise 49. Resistance of an Electrolyte.

When a constant current is sent through an electrolyte, polarization produces a counter E. M. F. and so renders the principle of Wheatstone's bridge inapplicable. Polarization may be avoided by the use of rapidly alternating currents. The simplest source of such alternating currents is a small induction coil of high frequency. The galvanometer must be replaced by an instrument capable of

detecting alternating currents. A telephone receiver of from 10 to 30 ohms resistance is best adapted for this purpose. Otherwise the manipulation is similar to that described above. Vary the resistances or move the contact maker until the sound in the telephone ceases or becomes a minimum.

The most convenient form of bridge is one which has the wire wound on an insulating cylinder and contact is made by a small grooved wheel rolling upon the wire as the cylinder is revolved. This form of bridge, (Fig. 63), is due to Kohlrausch.

Fig. 63.

It is evident that self-induction must be avoided as far as possible in all parts of the apparatus. The electrolyte is contained in a glass vessel furnished with two large platinum electrodes covered with platinum black. If the vessel have a cylindrical or prismatic form (Fig. 64), the specific resistance or better the conductivity may be readily determined from the formula given in Exercise 45. If the form of the vessel be irregular (Fig. 65), comparison must be made with an electrolyte of known conductivity k. As such a standard we may take a saturated solution of NaCl, sp. gr. 1.201 at 18° C. For such

Fig. 64.

Fig. 65.

a solution k $=$ 214 $\times$ 10^{12} [1 + 0,002 (t — 18)] c. g. s. units, or $=$ 21.4 $\times$ 10^{20} [1 + 0.002 (t — 18)] ohms. Letting the resistance of a standard NaCl solution be R and that of the electrolyte R_x, then $\frac{R}{R_x} = \frac{k_x}{k}$, or k_x, the conductivity sought is given by the relation

$$k_x = k . \frac{R}{R_x} . \qquad (75)$$

The quantity Rk is a constant for each vessel and is called the "*resistance capacity*" of the vessel.

Frequently, especially in German tables, the specific conductivity is referred to that of mercury at 0° C. Since the specific resistance of mercury at 0° C. is $\frac{1}{10630}$ ohms, $=$ $\frac{10^9}{10630}$ c. g. s. units, its *specific conductivity* is 0.00001063 c. g. s. units. To find k in terms of mercury at 0° C. multiply the value found from (75), by 94074.

The "molecular conductivity" μ, is defined by the equation $\mu = \frac{k}{m}$, where m is the number of gram molecules * of the substance dissolved in one liter of the solvent, at 18° C., and is frequently used in measurements of this kind.

FORM OF RECORD.

Exercise 49. To determine the specific and molecular conductivities of a solution of a salt, using three different dilutions.

Date

Preliminary:					Final measurements.						
R	a_1	a_2	r_2	t	Solution	a_1	a_2	r_2	R_x	k_x	μ_x
					m						
					m						
					m						

* A gram molecule of a substance is a mass of the substance, in grams, equal to the molecular weight of the substance.

Measurement of Electromotive Force and Potential Difference.

The simplest method for measuring a difference of potential between two points is to connect them to a voltmeter, or to any instrument whose graduation allows us to read off the potential difference in volts at its terminals. The voltmeter was employed for this purpose in Exercise 43. The student should bear in mind that the voltmeter must at all times possess a very high resistance R_v, if its indications are to be reliable. Thus in measuring the drop in potential over a known resistance R, the instrument actually gives not $E = I.R$, but $E' = I.\frac{R.R_v}{R + R_v}$. Now it is clear that E' can equal E only when R_v is so large that R may be neglected in comparison.

Exercise 50. Measurement of the E. M. F. of a Cell.

The apparatus (Fig. 66), consists of a battery B, of E. M. F. higher than that of the cell to be measured, a resistance R_1, and a voltmeter. The cell B', whose E. M. F. is to be determined, is joined in parallel with the voltmeter, so that the E. M. F. has the same direction as that of the cell B. Close key K and adjust the resistance R_1, until on closing key k, the reading of the voltmeter does not change. The reading gives us directly the E. M. F. of the cell B'.

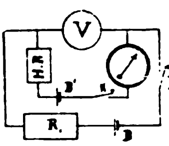

Fig. 66.

To increase the accuracy of the method a galvanometer more sensitive than the voltmeter may be inserted in series with B' and R_1 adjusted until the galvanometer shows no deflection on closing k. The high resistance H. R., may be used at first to prevent the passage of too

large a current through the cell, but it should be removed before making the final adjustment.

FORM OF RECORD.

Exercise 50. To measure the E. M. F. of five different cells, first separately and afterward all joined in series.

Voltmeter No.	Date ....
Name of cell	Voltmeter reading ....

Exercise 51. Electromotive Force by Potentiometer Method.

In measurements of E. M. F. or of potential difference where a high degree of accuracy is required, it is necessary to compare the E. M. F. to be measured with that of a standard cell.

The arrangement of the apparatus is shown in Fig. 67. The principle of the method consists in producing by means of a current from the battery B, a potential difference at the terminals of a resistance R, equal to the E. M. F. of the cell B′ whose electromotive force is to be measured. The two electromotive forces should be so arranged as to oppose each other in the galvanometer branch. If now the keys K and k be closed, no current will flow through the galvanometer and E = i R. In practice a standard cell is first placed at B′. Let its known E. M. F. be E_1; then $E_1 = i_1R_1$. The cell under experiment is next set in at B′, and after the resistance has been so adjusted that no current flows through the galvanometer on closing the keys, we have

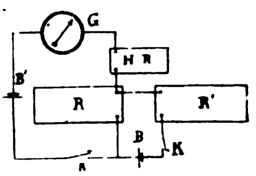

Fig. 67.

$$E_2 = i_2R_2, \qquad (76)$$

and under the condition that $i_1 = i_2$, we have at once

$$E_2 = E_1 \cdot \frac{R_2}{R_1}. \qquad (77)$$

The condition $i_1 = i_2$ is fulfilled when the battery B has a constant E. M. F., *and when the sum of the resistances R and R′ remains constant throughout the experiment.* It is moreover apparent that the E. M. F. of B must be greater than that of B′. A pair of Daniell cells or a storage battery is best for this purpose. If the E. M. F. of ordinary open circuit cells is to be measured, a freshly charged Leclanche element will do very well as the high resistance of R + R′ prevents it from polarizing appreciably, if the key K is kept closed for but an instant. The high resistance H. R., is inserted in the galvanometer circuit to prevent the passage of large currents through the instrument. It should be cut out when a balance is nearly obtained. The boxes R and R′ are 10000 ohm boxes and the sum of their resistances is kept at 10000 ohms.

It is clear that a wire of constant length and uniform cross-section may be substituted for the two resistances R and R′, and a balance obtained by shifting one terminal of the galvanometer along the wire. If the readings are a_1 and a_2 in the two cases, then

$$E_2 = E_1 \cdot \frac{a_2}{a_1}. \tag{78}$$

In this case however, care must be taken to avoid heating the wire, and to this end the key K should be kept closed but an instant at a time.

FORM OF RECORD.

Exercise 51. To measure the E. M. F. of five cells.

Date..................

	Name of cell.	R	R′	E
Galvanometer..........				
Room..........	....	....	....	....
Resistance boxes....	....	....	....	....
Standard cell No....	....	....	....	....
Temperature..........	....	....	....	....

Exercise 52. Calibration of a Voltmeter.

This method also consists in balancing the E. M. F. of one or more standard cells against the potential difference at the terminals of a resistance traversed by a current. Let V be the difference of potential at the terminals of the voltmeter, E be the E. M. F. of the standard cells, and i the current flowing through R and R′; then

$$V = i\,(R + R') \text{ and } E = i\,R\,, \tag{79}$$

or

$$V = E\,.\,\frac{R + R'}{R}\,. \tag{80}$$

The arrangement is that shown in Fig. 68. The resistance r is introduced to vary the readings of the voltmeter, but it should be noted that the voltmeter reading will then vary more or less with the change of R or R′. If the accuracy of any individual reading of the instrument is to be tested a repeated adjustment of r may be necessary. In this case it would be more convenient to adjust the number of cells in B so as to give, as nearly as possible, the desired reading or to keep the sum of R and R′ constant, which in general is not necessary.

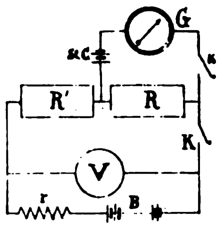

Fig. 68.

The voltmeter must be kept in the circuit all the time during the experiment. A high resistance in the galvanometer branch is also advisable here. The voltmeter correction is the amount to be added to the voltmeter reading to make it correspond to the computed value of V.

FORM OF RECORD.

Exercise 52. To calibrate voltmeter No. ....

Date ....

	Reading of voltmeter	R	R'	V	Correction
Galvanometer					
Room					
Standard cell No.	....	......	......	......	....
Resistance boxes..............	....	......	......	......	....
Temperature	....	......	......	......	....

Electromotive Force and Internal Resistance of Batteries.

Whenever a battery is closed through a conductor and no external work is done except that of heating the conductor, the value of the current is given by

$$E = I(R + r), \qquad (81)$$

where E is the electromotive force of the battery, I the current, R the external resistance and r the internal resistance of the battery. In case polarization occurs, the counter E. M. F. of polarization must be subtracted from E. According to the above formula the E. M. F. of the battery may be considered as divided into two parts: (a) IR, the drop of potential over the external resistance, commonly called *the terminal potential difference*, and (b), Ir, the drop in potential in the cell itself. The relative potentials of the various parts of the circuit may be represented as

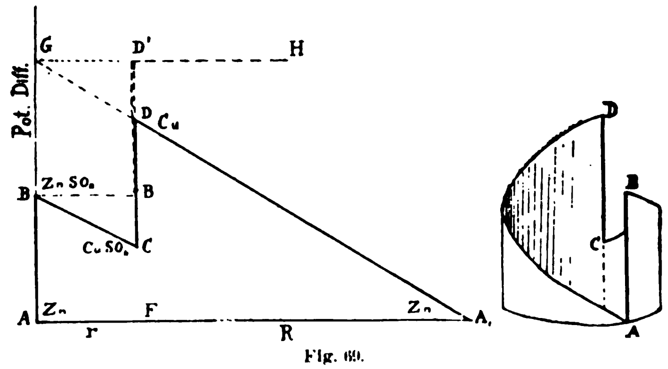

Fig. 69.

in Fig. 69, where the potentials are given as ordinates and the resistances as abscissae.

Consider the case of the Daniell cell. The potential of the zinc being the lowest in the whole circuit is represented by the point A. Between the *Zn* plate and the $ZnSO_4$ solution there is a sudden rise in potential of 0.52 volt, so that the potential of the $ZnSO_4$ solution next to the *Zn* plate is represented by the point B. Then follows a drop in potential in the cell until the Cu electrode is reached when a second sudden rise of 0.58 volt occurs, the potential of the $CuSO_4$ next to the copper plate, and that of the Cu electrode being represented by the points C and D respectively. From the copper plate, the potential falls off regularly owing to the external resistance R until the zinc plate is again reached at A_1.

To complete the analogy, the figure should be considered as wound upon the surface of a cylinder so as to make A_1 coincide with A. If both solutions are normal there is no appreciable difference of potential between the the liquids themselves. The electromotive force E, of a cell is the sum of the potential differences at the plates, i. e., AB + CD = AG, while the value of the terminal potential difference E′, is represented by FD, and depends upon the relative values of the external and internai resistances, R and r. In general this relation may be expressed thus :

$$E : E' :: R + r : R \qquad (82)$$

whence

$$r = R . \frac{E - E'}{E'} . \qquad (83)$$

From the above equations it is evident that E′ equals

E only when R is infinitely great, and E' is zero for R equal to 0. In measuring the E. M. F. of a battery by means of a voltmeter it must be noted that the voltmeter measures only the difference of potential at its terminals, and that its resistance must be very large in comparison with r, if it is to give reliable values for E.

Exercise 53. To Determine Terminal Potential Difference of a Cell as a Function of the External Resistance.

Connect a voltmeter or galvanometer of high resistance to the terminals of a Daniel cell B, as shown in Fig. 70. The reading will give practically the E. M. F. of the cell or a deflection which is proportional to it. Vary the parallel resistance R, by steps as follows: 60, 40, 20, 10, 6, 4, 2, 1, and 0.5 ohms, and observe the corresponding values of E'. Plot E' and R as shown in Fig. 71. The curve will be an hyperbola, if r remains constant.

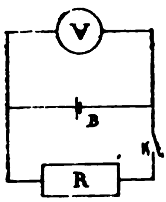

Fig. 70.

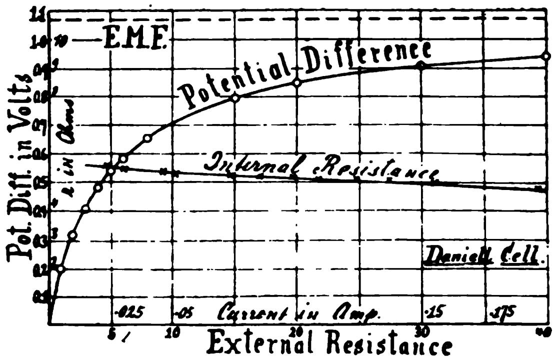

Fig. 71.

Why? The resistance of the cell r, may be found directly from the curve, for r = R when E' = E/2. Calculate also the value of r from equation (83), and plot r as a function of I, which equals E/R'.

FORM OF RECORD.

Exercise 53. To determine the terminal potential difference of a Daniell cell as a function of the external resistance.

Voltmeter No. .... Room .... Date ....

Resistance box	R	E′	r	I
E (for R infinite)	....	....	....	....
	....	....	....	....

Exercise 54. To Determine Electromotive Force and Internal Resistance of Five Different Cells by Voltmeter and Ammeter.

This method is a modification of the preceding. A milliammeter is joined in series with the resistance box R, as in Fig. 70. The reading for E is made with the key open and then readings for E′ and I are taken simultaneously, with K closed. Since $\frac{E'}{R} = I$ we have

$$r = \frac{E - E'}{I}. \tag{84}$$

Vary the resistance of the ammeter circuit and take three different currents for each cell, but all of such a value that E′ remains nearly equal to E/2. If R and the resistance of the ammeter be known, formula (83), may be used as a check formula, giving check values r′, which should agree with those of r. In the case of dry cells the current is frequently so small that it can not be read accurately. In this case compute r from the values of E, E′, and R.

FORM OF RECORD.

Exercise 54. To determine electromotive force and internal resistance of five different cells by voltmeter and ammeter.

Voltmeter No.	Name of cell	E	E′	I	R	r	r′
Ammeter No.	....				-- ammeter		
Resistance box	....				....		

Since the polarization is quite rapid in some cells, it is better to read E immediately after the key K is opened, than at the beginning of each observation.

Exercise 55. Electromotive Force and Internal Resistance of Five Cells by Condenser Method.

The foregoing method has two disadvantages: first the voltmeter reading is only approximately equal to the E. M. F. of cells of low internal resistance, and second the key K must remain closed until the readings can be taken, thus permitting of considerable polarization. In the condenser method the first objection is entirely overcome and the time needed for the observation is much shorter than in the previous method. The principle of this method is to obtain a deflection of a ballistic galvanometer proportional to the E. M. F. or difference of potential of the cell.

Two methods may be employed. (a) As shown in Fig. 52, a condenser of capacity C, and a ballistic galvanometer are substituted for the voltmeter in the last exercise. The condenser is charged to the difference of potential to be measured and discharged through the ballistic galvanometer. (b) The apparatus is arranged as in Fig. 72. In this case the condenser is charged through the galvanometer, the resulting deflection being proportional to the electromotive force charging the condenser. To obtain E the key K should be open; to determine E′, it should be closed. The advantage of this method consists in being able to take the reading at the instant the circuit is closed.

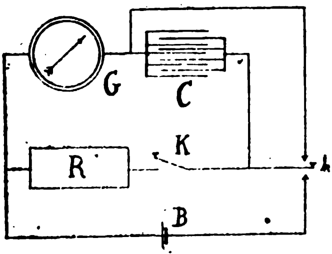

Fig. 72.

In order to express the E. M. F. in volts, the condenser is also charged from a standard cell and discharged

through the galvanometer. Let the deflection in this case be d_2, and let d be the deflection given by the cell under investigation when on open circuit, and d_1 when the cell is closed through the resistance R. The deflections are proportional to the quantities of electricity passing through the galvanometer, and these are themselves equal to the product of the difference of potential at the terminals of the condenser, by the constant capacity, C. Therefore if E_s be the E. M. F. of the standard cell, then

$$E = E_s \cdot \frac{d}{d_2}, \tag{85}$$

and

$$r = R \cdot \frac{d - d_1}{d_1}. \tag{86}$$

The charging and discharging may be done in a small fraction of a second and the key K, need only be held down during that time in the determination of E′.

It should be observed that the internal resistance of batteries, and especially of dry batteries is not a constant, but seems to decrease with a decrease of R, that is with increasing current furnished by the cell. This is probably due to polarization, whose effect is neglected in the formulæ given above.* Use three different resistances for each cell, such that d_1 is about d/2.

FORM OF RECORD.

Exercise 55. To determine electromotive force and internal resistance of five cells by the condenser method.

Galvanometer .... Room .... Date ....

	Name of cell	d	d_1	R	E	r
Condenser	....	....	....	....	....	....
Resistance	....	....	....	....	....	....
Deflection by st. cell	....	....	....	....	....	....
Temp. of st. cell	....	....	....	....	....	....
E. M. F. of st. cell	....	....	....	....	....	....

* Guthe, Physical Review, VII, p. 103, 1898.

Exercise 56. Internal Resistances of a Cell. (Method of Nernst and Haagn.)

In this method the internal resistance of the cell is determined by means of an alternating current, while the cell itself furnishes no current whatever. In this way the disturbing effects of polarization are entirely avoided. The alternating current is furnished by the induction coil I C., (Fig. 73), which charges the condenser C alternately to positive and negative potentials. These alternations are transmitted to a Wheatstone bridge, of which the condensers C_1 and C_2 form two arms, and the resistances R and r form the other two, where r is the resistance of the battery B. The resistance R is so adjusted as to produce minimum sound in the telephone receiver which replaces the galvanometer in the ordinary Wheatstone bridge.

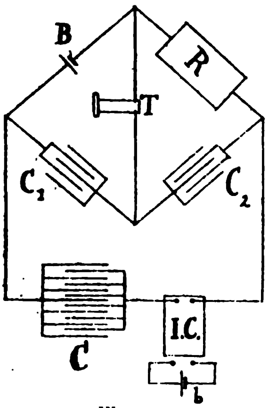

Fig. 73

The applied difference of potential produces a current i, through the resistances R and r. Condenser C_1 will be charged with a quantity $C_1 i r$, and condenser C_2 with a quantity $C_2 i R$. For a minimum sound in the telephone these charges are equal or

$$C_1 i r = C_2 i R ,$$

or

$$r = \frac{C_2}{C_1} R .$$

In practice it is best to connect a slide-wire bridge in series with the battery and connect the telephone with the

contact maker. We have then instead of r in one arm of the Wheatstone bridge, r + R′, where R′ is the resistance of the part of the wire from the battery B to the contact maker. The formula becomes in this case

$$r = \frac{C_2}{C_1} R - R' .$$

FORM OF RECORD.

Exercise 56. To determine the internal resistance of five cells by the method of Nernst and Haagn.

Name of cell	C_1	C_2	R	R′	r

Measurements of Current.

Since it is obviously impossible to preserve standards of current, it is also impossible to determine currents by direct comparison with a standard as in the case of resistance or electromotive force. It is necessary therefore to employ certain known effects of the current for purposes of measurement. The following are those chiefly used in laboratory practice. (a) Electromagnetic effect in galvanometers, voltmeters and ammeters. These instruments have been in frequent use for the measurement of currents in the previous experiments. If any doubt as to their accuracy arises they should be calibrated by one of the methods which follow. (b) Chemical effect. Copper or silver voltameter, (Exercise 57). (c) The production of a potential difference at the terminals of a known resisttance, (Exercise 58).

Exercise 57. To Calibrate an Instrument by the use of the Copper Voltameter.

According to Faraday's law the quantity of a substance deposited by an electric current is proportional to the

quantity of electricity passing through the electrolytic cell. A steady current of one ampere will deposit 4.025 grams of silver or 1.1838 grams of copper in one hour. The electrochemical equivalent z, of a substance is the number of grams of the substance deposited by unit quantity of electricity. Since the quantity of electricity is the product of current and time it is easy to determine the *average current* flowing through an electrolytic cell, by dividing the number of grams of the substance deposited in time t, by the weight which would have been deposited by unit current in the same time. Thus if w be the number of grams deposited, then

$$I = \frac{w}{z\,t}. \qquad (87)$$

Whenever it is desired to calibrate an instrument for measuring current by means of the voltameter it is necessary to determine the average reading of the instrument during the time the current flows, and by comparing this reading with the average current, to determine the constant of the instrument or the correction to be applied in the case of direct reading instruments.

TREATMENT OF THE COPPER VOLTAMETER.

Of the various forms of voltameters the simplest is the copper voltameter. (Fig. 74). Its manipulation demands considerable care. The voltameter consists of two copper electrodes in the form of plates or spiral wires, immersed in a solution of $CuSO_4$. This solution must be kept in a separate bottle and be used for this experiment only. * That part of the kathode on which the copper is

* The following solution is recommended: $CuSO_4$, 15 grams; H_2SO_4, 5 grams; alcohol, 5 grams; water, 100 grams.

to be deposited must be kept perfectly clean and ***must not be touched with the fingers.*** If not clean, the plate must be dipped into a strong solution of potassium cyanide then washed well with water and then dipped into strong alcohol and dried. All these operations should be performed as rapidly as possible since moist copper oxydizes easily in the air. In case the kathode is perfectly clean it may be used without further preparation.

Fig. 74.

First weigh the kathode to 0.1 milligram. Then set up the voltameters, two in series, in order to check the result. In the circuit place a battery of high, and if possible, constant E. M. F., a large variable resistance, the instrument to the calibrated, and a key for opening and closing the circuit. The resistance should be adjusted beforehand so as to give the current that is to be sent through the voltameters. The current density may be as high as 1.5 amperes per square-decimeter of the electrodes. Close the circuit for a definite time, say thirty minutes, and note the reading of the instrument every minute. After the circuit is opened wash the kathode in plenty of water, rinse in alcohol, dry and weigh to determine the mass of the deposit.

FORM OF RECORD.

Exercise 57. To calibrate a .... by copper voltameter.

Date ....

Name and number of instrument .... Formula of instrument ....

	Voltameter I.	Voltameter II.	t	Reading
Wt. of kathode before	....	....	....	....
Wt. of kathode after	....	....	....	....
Gain	....	....		

Average gain .... Average reading of Inst. ....

Average current .... Constant (or correction) of Inst. ....

Exercise 58. Calibration of Ammeter by Standard Cell.

A current may be measured by comparing the difference of potential at the terminals of a known resistance r, through which it flows, with a known E. M. F. Let A (Fig. 75), be an ammeter to be calibrated; B a battery of constant E. M. F., and s an adjustable resistance for varying the current. R_1 and R_2 are two resistances, each very large in comparison with r. The standard cell is set in such a way as to oppose the difference of potential at the terminals of R_1. The resistances R_1 and R_2 are so adjusted that on closing k after K, no current will flow through the galvanometer; then

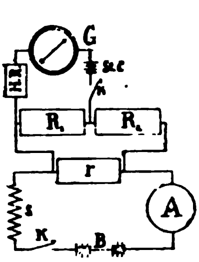

Fig. 75.

$$I\,r = i\,(R_1 + R_2) \tag{88}$$

where I is the current flowing through r, and i that through R_1 and R_2. I is sensibly the same current as that through the ammeter.

Moreover $\qquad E_s = i\,R_1, \qquad (89)$

therefore

$$I = \frac{E_s}{r} \cdot \frac{R_1 + R_2}{R_1} \tag{90}$$

It is apparent that under the conditions given the current must be equal to or greater than E_s/r in order to make a balance possible. * Calibrate an ammeter using at least five different currents.

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* For a method for measuring small currents see Carhart and Patterson, p. 172.

FORM OF RECORD.

Exercise 58. To calibrate ammeter No. ....

Date......... ....

Zero point of ammeter.......... Room .... Resistance r.......

Standard cell No.. .... Temperature ...... E. M. F. .. ..

Ammeter reading		R_1	R_2	I computed	Correction.
Observed	Corrected for zero pt.	....	....	....	....
....	.. ..	....	.. ..	.. ..	....
....	....	.. ..	....	.. ..	....

Comparison of Capacities.

Exercises 59. Comparison by Direct Deflection.

The simplest method of determining the capacity of a condenser consists in comparing the deflections of a ballistic galvanometer (Fig. 76), caused by the discharge of the quantities of electricity stored in a standard condenser and in that under investigation, when each has been charged to the same difference of potential. From Exercise 41,

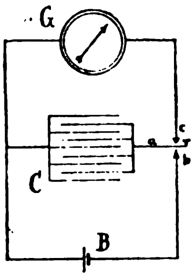

Fig. 76.

$$d_1 = Q_1/c = aE\,C_1 \qquad (91)$$

$$d_2 = Q_2/c = a\,E\,C_2 \qquad (92)$$

$$C_2 = C_1 \cdot \frac{d_2}{d_1} \qquad (93)$$

One of the condensers C_1, is a standard condenser of known capacity.

If the capacity of a condenser is so large as to give too large deflections, when discharged through the galvanometer, the experiment may be arranged as shown in Fig. 77. A battery of constant E. M. F. sends a current through two resistances R_1 and R_2, the sum of which must be kept constant. The condenser is connected over one of these resistances R_1, and charged

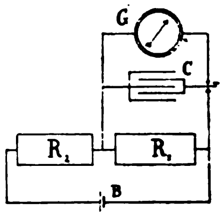

Fig. 77.

with the difference of potential at its terminals equal to i R_1. If the deflection on discharging is too large the value of R_1 is to be reduced, R_2 being increased by the same amount. Then if C_1 and C_2 represent, respectively, the standard and the unknown capacity

$$d_1 = a\ i\ R'_1C_1 \text{ and } d_2 = a\ i\ R''_1C_2\ , \qquad (94)$$

therefore

$$C_2 = \frac{d_2\ R_1'}{d_1\ R_1''} \cdot C_1 \qquad (95)$$

Instead of two resistance boxes the wire of a slide wire bridge may be used. Not unfrequently condensers are found whose capacity depends very largely upon the time of charge. Such condensers are termed "absorbing condensers." The student should therefore take different intervals for charging and note carefully the resulting effect upon the observed capacity.

FORM OF RECORD.

Exercise 59. To measure the capacities of four condensers.

Room .... Date ....

Galvanometer	Name of condenser	Time of charge	R_1	d	C
Resistance boxes	....	....			
Cell	....	....			

Exercise 60. Comparison of Capacities by the Method of Mixtures.

The apparatus is arranged as shown in Fig. 78. In the middle of the figure is shown a Pohl's commutator with the cross wires removed, by means of which the points a and a′ may be connected either to b and b′ or to c and c′. By means of the first connections the condensers C_1 and C_2 are charged with quantities of electricity equal to R_1iC_1 and R_2iC_2 respectively. On changing the connections of a and a′ these

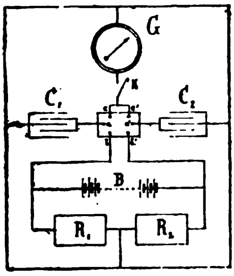

Fig. 78.

quantities mix and there will, in general, be a deflection of the galvanometer resulting from the difference of these two charges on pressing the key K. Adjust the resistances R_1 and R_2 until there is no deflection of the galvanometer; then

$$R_1 i C_1 = R_2 i C_2 \tag{96}$$

and

$$C_2 = C_1 \cdot \frac{R_1}{R_2} \tag{97}$$

The battery should have a relatively high E. M. F.: six to ten Lechanche elements may be used. The disturbing influence of absorption may be studied as in the preceding experiment.

FORM OF RECORD.

Exercise 60. To compare two condensers by the method of mixtures.

Room .... Date ....

	Name of condenser	Time	R_1	R_2	C
Galvanometer					
Resistance boxes	....	....			
Standard condenser......	....				

Measurement of Inductance.

Exercise 61. Comparison of Selfinductance of a Coil with Standard of Selfinductance.

Arrange the apparatus as in Fig. 79, in which the two selfinductances each form one arm of a Wheatstone bridge. Let their resistance and selfinductance be R_1 and L_1, R_2, and L_2 respectively. The remaining two arms are formed by the non-inductive resistances R_3 and R_4. First adjust R_3 and R_4 for *constant current* until the galvanometer shows no deflection on closing k, that is, $\frac{R_1}{R_2} = \frac{R_3}{R_4}$. Then, *keeping k closed*, open K. There will in

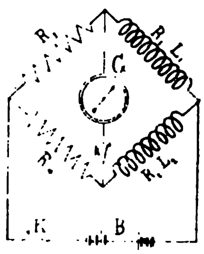

Fig. 79.

general be a deflection of the galvanometer due to the difference of the E. M. F. produced by the selfinductances in the branches L_1 and L_2. Vary the selfinductance of the standard (see page 115), until upon opening K no deflection is obtained. Then the points connected to the galvanometer will be at the same potential as well during the steady as during the variable state of the current.

Let the difference of potential over R_1 or R_2 for a constant current be E; then the current through R_1 is $i_1 = \frac{E}{R_1}$, and through R_2, $i_2 = \frac{E}{R_2}$. Now the value of the E. M. F. due to selfinductance is

$$e = - L \cdot \frac{di}{dt} \cdot \text{*} \qquad (98)$$

In the case under consideration, the value of e on opening K at any moment, is

$$e_1 = - L_1 \cdot \frac{di_1}{dt} = - \frac{L_1}{R_1} \frac{dE}{dt} \qquad (99)$$

and

$$e_2 = - L_2 \cdot \frac{di_2}{dt} = - \frac{L_2}{R_2} \frac{dE}{dt} \cdot \qquad (100)$$

But after the adjustment of the standard, $\frac{dE}{dt}$ is the same for both e_1 and e_2, and hence the condition for no deflection during the variable state of the current is given by the equation

$$\frac{L_1}{R_1} = \frac{L_2}{R_2} \text{ or } L_1 = L_2 \frac{R_1}{R_2} = L_2 \frac{R_3}{R_4} \cdot \qquad (101)$$

In practice care must be taken to place the two selfinductances in such a position that they will not influence each other. To obtain accurate results the battery must

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* Carhart, University Physics, Vol. II, p. 383.

possess a high voltage. The best results are obtained by using an alternating current and substituting for the galvanometer a telephone receiver as in Exercise 56, or by rectifying the current before sending it through the galvanometer. In an instrument designed by Ayrton and Perry, called the secohmmeter, * this rectification of the current is effected by means of a double commutator.

FORM OF RECORD.

Exercise 61. To measure the selfinductance of......

Date.................

Apparatus	....	R_3	R_4	Reading of Standard	L
Room	....	..	..	....	..
Voltage of battery	....	..	..	....	..

Exercise 62. To Measure the Mutual Inductance of two Coils.

According to definition the coefficient of mutual induction or, the mutual inductance of two coils is given by the equation

$$e = -M\frac{di}{dt},$$

where e is the counter E. M. F. in the secondary due to the mutual inductance M, and i the current in the primary. Let the induced E. M. F. produce a current through a ballistic galvanometer; the current i′, so produced, will at any moment be e/r, where r is the total resistance of the secondary circuit including the galvanometer. The total quantity of electricity passing through the galvanometer is

$$\int i'dt = \int\frac{e}{r}\,dt = -\frac{M}{r}\int_0^{I}di,$$ where the integral is to be taken between 0 and I, the final value of the steady cur-

*Carhart and Patterson, p. 110.

rent in the *primary circuit.* Therefore the total quantity in the secondary circuit when the primary circuit is closed, becomes, neglecting its direction,

$$Q = \frac{MI}{r}. \tag{103}$$

If the current through the primary be suddenly reversed the value of Q is doubled. From the formula for the ballistic galvanometer we have Q = cd, where c is the constant of the galvanometer. Therefore

$$d = \frac{M\,I}{c\,r} \text{ and } M = \frac{c\,r\,d}{I}. \tag{104}$$

The deflection is therefore proportional to the current in the primary and inversely proportional to the resistance of the secondary circuit. To show these relations clearly make the following experiments. (1) Vary I keeping r constant. Plot values of d and I.

(2) Vary r keeping I constant. Plot d and 1/r; both curves should be straight lines. The arrangement is shown in Fig. 80. Let P represent the primary and S the secondary coil. The primary is in series with a battery B, a variable resistance R, and an ammeter A. To the secondary coil are joined a resistance r and a ballistic galvanometer.

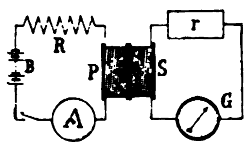

Fig. 80.

FORM OF RECORD.

Exercise 62. To determine the mutual inductance of......

Date ....

Ammeter .... Room .... Ballistic galvanometer..................

Determination of constant. (1). r constant. (2). I constant.

	I	d	M	r	d	M
Capacity						
Electromotive force	......	......	......	......	......	......
Deflection	......	......	......	......	......	......
Constant	......	......	......	......	......	......
	......	......	......	......	......	......

CHAPTER IX.

MAGNETIC MEASUREMENTS.

Magnetic Fields.

In most of the foregoing experiments the action between a magnetic field produced by a current and another magnetic field was used to determine electrical quantities. In the following experiments the subject will be approached from a different point of view and the student will find it of advantage to review the subject of galvanometers, pp. 117 to 120, before reading this chapter.

Exercise 63. Determination of H. (First Method).

The lines of magnetic force due to the earth's field run from South to North, although deviating in some places by an appreciable angle from the geographical North and South line. This angle is called the angle of declination. The lines of force are also inclined towards the horizontal plane, making in Ann Arbor an angle of 72° with the horizon. This is the angle of magnetic inclination or the "magnetic dip." It should be noted that neither the magnetic declination nor the dip are constant. They not only vary from place to place over the earth's surface, but they also vary slightly from year to year and from day to day in the same place. The magnetic field of the earth may be conceived as the resultant of two components, one

horizontal H, and one vertical V. Then θ, the angle of dip, is given by the equation

$$\tan \theta = \frac{V}{H} . \qquad (105)$$

It is of great interest to determine the value of the horizontal component of the earth's magnetism in terms of the fundamental units, of length, mass and time, since all the practical magnetic and electrical units are based on these.*

A relation between the magnetic moment of a magnet and the strength of the magnetic field in which it is situated may be derived in either one of two ways:

(*1*) *Method of deflections.*

Let a magnet N S, (Fig. 81,) be placed with its axis on the magnetic East-West line † and on this line in the same horizontal plane, at a measured distance from it place a very small magnetic needle n s, suspended by a fine silk thread or quarz fiber.

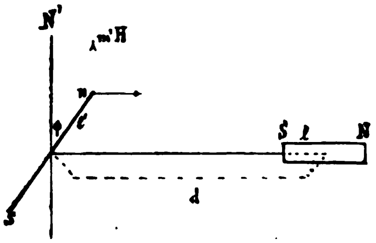

Fig. 81.

Let m and l be the pole strength and half-length of magnet N S. Let m′ and l′ represent the same for magnet n s.

Also let

d be the distance between the centers of the magnets,
H the horizontal component of the earth's magnetic field,
M the magnetic moment of the magnet N S, = 2ml,
M′ the magnetic moment of the magnet n s = 2m′l′.

* For the relations of these units to each other, see Carhart. University Physics. Vol. II, pp. 315 and 340-341.

† To find the magnetic East-West line, suspend in the center of a plane coil of wire a magnetic needle. Turn the coil until on sending a current through it the needle is not deflected. The plane of the coil is then in the magnetic East West line.

The magnet n s will be deflected from its normal position by the angle φ, such that the turning moments due to the earth's field and that due to the influence of the magnet N S are equal.* The force due to the earth's field on one of the poles of n s is m'H, and the lever arm on which it acts is, $l' \sin \varphi$; so the turning moment on the whole magnet is $2m'l'H \sin \varphi = M'H \sin \varphi$.

The force on the north-seeking pole of n s due to the south-seeking pole of N S is, according to Coulomb's law of the inverse squares, $\frac{m\ m'}{(d - l)^2}$, l' being considered negligible in comparison with d. The force of the north-seeking pole of N S is in the opposite direction and equal to $\frac{m\ m'}{(d + l)^2}$, so that the whole force on the north-seeking pole of the needle is:

$$F_1 = m\ m' . \left\{ \frac{1}{(d - l)^2} - \frac{1}{(d + l)^2} \right\} \tag{106}$$

and if l be small in comparison with d,

$$F_1 = \frac{4m\ m'l}{d^3} . \tag{107}$$

The turning moment due to this force is $F_1 l' \cos \varphi$, and since the turning moments on the two poles of n s are equal and in the same direction the total turning moment exerted on N S by the magnet n s is $2\ F_1 l' \cos \varphi$, or

$$\frac{8\ m\ m'\ l\ l'}{d^3} \cos \varphi = 2 . \frac{M\ M'}{d^3} \cos \varphi . \tag{108}$$

But the turning moments represented by the force

---

* The angle φ is determined by mirror and scale. Suppose the distance of the mirror from the scale to be D and the deflection d, then $\tan 2\varphi = d/D$.

exerted by the earth and the magnet N S must be equal since the needle is in equilibrium: So the expressions for these turning moments may be set equal to each other, or

$$M' H \sin \varphi = 2 \frac{M M'}{d^3} \cdot \cos \varphi \qquad (109)$$

whence

$$\frac{M}{H} = \frac{d^3}{2} \cdot \tan \varphi = A . \qquad (110)$$

(2) *Method of Oscillations.*

The law of vibration of a magnetic needle in a magnetic field is the same as that of the pendulum. If we suspend our deflecting magnet NS, so as to swing freely, its period is

$$T = 2 \pi \sqrt{\frac{K}{MH}} , \qquad (111)$$

where T is the period of a complete vibration and K is the moment of inertia of the magnet. In order to find K the same method is applied as in Exercise 20, by putting a brass ring of known moment of inertia K', on the magnet so that the axis of rotation and the axis of the ring coincide. Let the period of vibration of the new system be T', then

$$T' = 2 \pi \sqrt{\frac{K + K'}{M H}} , \qquad (112)$$

and from the equations 111 and 112

$$M H = \frac{4 \pi^2 K'}{T'^2 - T^2} = B . \qquad (113)$$

Equations 110 and 113 furnish two expressions involving M and H.

$\frac{M}{H} = A$ and $M H = B$, from which

$$H = \sqrt{\frac{B}{A}}. \tag{114}$$

or on substituting the values of A and B,

$$H = \frac{2\pi}{d}\sqrt{\frac{2K'}{d(T'^2 - T^2)\tan\varphi}}. \tag{115}$$

It is obvious that by this method the value of H may be determined in the fundamental units of mass, length and time.

The instrument used for this experiment is called a magnetometer. It consists of a closed glass case furnished with windows to enable the observer to measure the deflection of the needle by means of the mirror which it carries. Place the deflecting magnet at a certain distance from the small needle as described above and observe the deflection. Turn the magnet end for end and again observe the deflection, which will now be in the opposite direction. Repeat these two operations with the magnet at the same distance on the opposite side of the needle and take the mean of the four observations as the deflection φ. In case the length of the deflecting magnet is not negligible in comparison with the distance d, we may take it into account. Kohlrausch has shown that for a bar magnet the distance between the poles is very nearly 5/6 of the length of the magnet. So the formula for H becomes

$$H = 2\pi\sqrt{\frac{2K'd}{(T'^2 - T^2)(d^2 - l^2)\tan\varphi}} \tag{116}$$

In the vibration method the time of vibration may be determined as in Exercise 6, or by the use of an ordinary stop watch if less accuracy is required. The stop watch should however be compared with a standard clock. How may the magnetic moment of the deflecting magnet be determined from the foregoing formulae?

FORM OF RECORD.

Exercise 63. Determination of H.

Date ....

(1) Determination of $\frac{M}{H}$.

	Position of magnet.	Deflection	tan 2φ	tan φ
Distance d	a	....	......	....
Length of magnet	Reversed	....	......	....
(d l)	b	....	......	....
Distance D	Reversed	....	......	....

Computation of $\frac{M}{H}$

(2) Determination of M H:

(a) Determination of T.

(b) Determination of T′.

(c) Data for moment of inertia of ring.

Mass of ring .... Outer diameter of ring .... Inner..........

Moment of inertia of ring ....

Computation of M H.

(3) Computation of H.

Exercise 64. Determination of H. (Second Method).

In formula 111, $T = 2\pi\sqrt{\frac{K}{MH}}$

the torsional moment of the suspending wire was neglected. The complete formula for a given magnet is

$$T = 2\pi\sqrt{\frac{K}{MH + \mathcal{T}_1}} \quad \text{or} \quad \frac{1}{T^2} = H\,c. + c' \qquad (117)$$

where $\mathcal{T}_1$ is the moment of torsional couple of the suspending wire, and c and c′ are constants. These relations are illustrated in the following experiment. Let a short magnetic needle be suspended at the center of a solenoid whose length is at least twenty times its diameter, and whose axis is parallel to the magnetic meridian. Observe the period of vibration of the needle, first when swinging in the earth's field, and then when a magnetic field due to a known current I through the solenoid, is superposed upon the field of the earth. Let the period of vibration of the system in the first case be T and in the second T_1. Then

$$\frac{1}{T^2} = H\,c + c' \qquad (118)$$

$$\frac{1}{T_1^2} = c\,(H + H_1) + c', \text{ etc.}$$

Vary the current and observe T_2, T_3, T_4, etc.; also the periods of vibration T'_1, T'_2, T'_3, etc., when the current through the solenoid is reversed. Then plot $1/T^2$ as ordinates and the magnetic field strength produced by the currents as abscissae, the latter positive or negative according to direction.* The curve consists of two straight

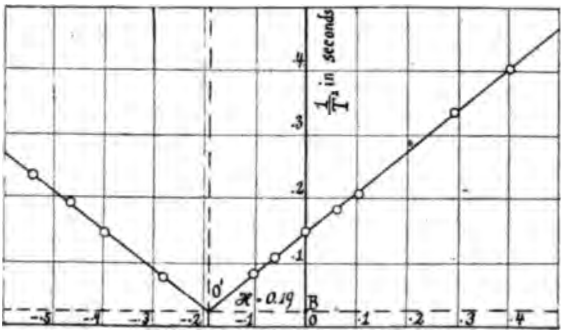

Fig. 82.

lines meeting at the point O′, Fig. 82 The time corresponding to the ordinate O B is then the period of vibration corresponding to the rigidity of the wire alone. If O′ be considered as the origin and the strengths of field be plotted on O′x it is evident that O′B is the value of the horizontal component of the earth's field, and that the strength of any field may be readily determined by allowing the magnet under experiment to vibrate in this field and determining its period of vibration. Then

$$H = \frac{1}{T^2 c} - \frac{c}{c}, \tag{119}$$

where c and c′ are to be substituted from the foregoing observations.

---

* If the current be increased in the negative direction beyond a certain value the magnet turns through 180°. Why?

In practice a storage cell of constant E. M. F. is used to furnish the current through the solenoid. A resistance is joined in series to allow the current to be varied within wide limits. The periods of vibration may be determined by means of a stop watch. It may be shown theoretically that the strength of field at the center of a long solenoid carrying a current I, is

$$H = \frac{4 \pi n I}{10 l} * \qquad (120)$$

where n is the number of turns of wire and l the length of the solenoid. The current I is computed from the electromotive force of the cell and the total resistance.

FORM OF RECORD.

Exercise 64. Determination of H. (Second Method).

Date..............

Dimensions of solenoid :	R	I	T	$\frac{1}{T^2}$
Length..................	....	....	....	
n = ...	....	....	....	.....
Determined from curve :	....	....	....	.....
c = ...	....	....	....	.....
c′ = ...	....	....	....	.....
H = ...	....	....	....	.....

Magnetic Properties of Iron and Steel.

When a bar of unmagnetized iron or steel is introduced into a magnetic field the number of lines of force is greatly increased. This becomes especially apparent when the iron is introduced into a solenoid carrying a current I. Before the introduction of the iron the strength of the magnetic field in the solenoid is

$$H = \frac{4 \pi n I}{10 l} \qquad (121)$$

* Carhart, University Physics, Vol. II, pp. 365-366.

After the introduction of the iron the field strength rises to a much higher value. The number of lines of force per cm.[2] is in this case usually designated by the letter B, and is called the "*flux density.*" The "magnetic permeability" μ, is the ratio between the number of lines of force per cm^2. in iron and in air; it is given in mathematical form by

$$\mu = \frac{B}{H}. \qquad (122)$$

Exercise 65. Magnetization Curve for Iron and Steel.

It is of interest to determine the dependence of B upon the field strength H, produced by the current alone, when the field is slowly increased from 0 to 30 or 40 lines per cm^2. It will be seen from Fig. 83, that B increases slowly at first, then very rapidly, then slowly again until it finally increases at the same rate as H; that is, the presence of the iron does not introduce any additional lines of force. The iron is then said to be saturated.

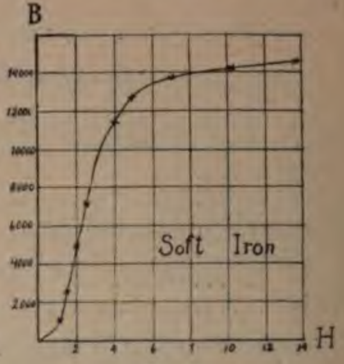

Fig. 83.

The method here described is known as the ballistic or ring method. The metal is given the form of a ring of uniform cross section, Fig. 84. It is best to have the cross section of the ring approach the form of a rectangle and the diameter large as compared with the thickness. The outer and inner diameters of the ring must be measured with care, and the volume of the ring obtained from its loss of weight in

Fig. 84.

water. The cross section A is readily found from the volume by dividing by the average diameter. The ring is then covered with insulating tape upon which is wound a layer of wire covering the entire ring and forming the primary coil. The number of turns n_1, in the primary coil must be carefully determined. Over this is wound the secondary coil, five to twenty turns, n_2. The apparatus should be arranged as shown in Fig. 85. In the primary circuit are joined in series a storage battery, an ammeter, a rheostat R, in which the resistance may be suddenly varied, the primary coil of the ring and a commutator for reversing the current through the coil. The secondary coil is connected to a ballistic galvanometer whose constant is known.* If we vary the number of lines of magnetic force in the solenoid, the electromotive force induced in each turn of the secondary is at any moment equal to the rate of change of the number of lines of force threading through the circuit, or

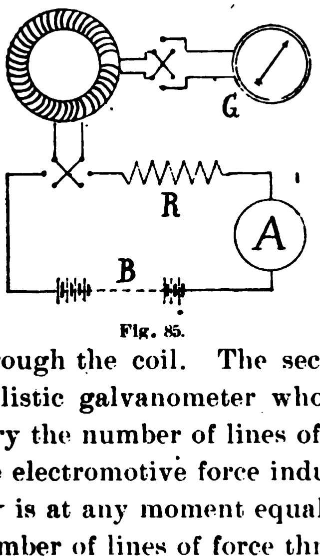

Fig. 85.

$$e = -\frac{dN}{dt}. \qquad (123)$$

If there are n_2 turns of wire in the secondary coil then

$$e = -n_2 \cdot \frac{dN}{dt}. \qquad (124)$$

The current i, passing through the galvanometer is then

* As the damping of the galvanometer may be different on open and closed circuit, it is best to short circuit the galvanometer through a suitable external resistance immediately after the discharge of a condenser.

$$i = -\frac{n_2}{r_2}\frac{dN}{dt}, \tag{125}$$

where r_2 is the total resistance of the secondary circuit. The total quantity of electricity Q, passing through the ballistic galvanometer is $\int i\,dt$, corresponding to a change of N lines of force, or

$$Q = \frac{n_2 N}{r_2} = c\ d, \tag{126}$$

where c is the constant of the galvanometer and d the throw. Express c in c. g. s. units. Since the constant is usually given in micro-coulombs per scale unit, one micro-coulomb being 10^{-7} c. g. s. unit, and the resistance in ohms, one ohm being equal to 10^9 c. g. s. units, the formula becomes

$$N = 100.\ \frac{c\ d\ r_2}{n_2}. \tag{127}$$

Since B denotes the number of lines of force per unit area the increase of B corresponding to a deflection d of the galvanometer when the current is varied in the primary, ΔB, is given by

$$\Delta B = \frac{N}{A} = 100\ .\ \frac{c\ d\ r_2}{n_2 A}. \tag{128}$$

To obtain the magnetization curve it is important to start with an unmagnetized ring. It is best to demagnetize the ring by sending first a rather large current, say three amperes, through the primary, increasing the resistance R, and then reversing the direction of the current. This must be done several times, taking care to decrease the current each time before the reversal, until the current has become very small. On breaking the circuit the ring will contain no residual magnetism. A more convenient way is to connect the primary to the terminals of a small

alternator driven by a belt. Let the alternator attain its full speed and then throw off the belt. As the speed decreases the current decreases, and the iron is as before subjected to cycles with constantly decreasing current.

Arrange the apparatus, as shown in Fig. 85. Adjust the resistance so as to give a small current through the primary coil. Reverse the current and observe the first ballistic throw of the needle d'_1, reverse again and observe d''_1; the mean d_1, corresponds to a reversal of the current i, read from the ammeter. This is repeated, increasing the current step by step, until the deflections increase very slowly with increasing current. Compute H from formula 121, and B from 128, remembering that but *half the average deflection must be taken*, since the current is changed each time by 2 i instead of by i.

Plot B and H; the curve will resemble that shown in Fig. 83. From the curve it appears at once that μ is not a constant, but varies greatly with the degree of magnetization.

FORM OF RECORD.

Exercise 65. Magnetization curve for iron.

Date.............

Galvanometer...... Room...... Ammeter No.......
Constant of galvanometer. Resistance r_2..........
Ring.... Outer diam..... Inner.... Average....

	I	d'	d''	d	d/2	H	B	μ
Volume of ring....								
Cross section......								
Turns in primary..								
Turns in secondary								

Exercise 66. Hysteresis Curve for Iron or Steel.

If instead of reversing the current in the previous exercise we should decrease I by small steps, observe the corresponding throws of the galvanometer and plot B with

respect to H, we obtain an entirely new curve owing to the effects of "residual magnetism" in the iron or steel. The experiment thus allows us to study the lagging of the induced magnetism behind the magnetizing force. Thus if, in the beginning, the iron were well magnetized, the starting point on the curve would be the highest point to the right, (Fig. 86.), and on reducing the current to 0, there would still remain in the iron the lines of force represented by the positive ordinate. The iron is still magnetized, but on reversing the current the value of B decreases very rapidly, becomes 0 for a small negative current, and finally reaches a value symmetrical to that at the starting point, when the current in the negative direction reaches the same value it had at the beginning in the positive direction.

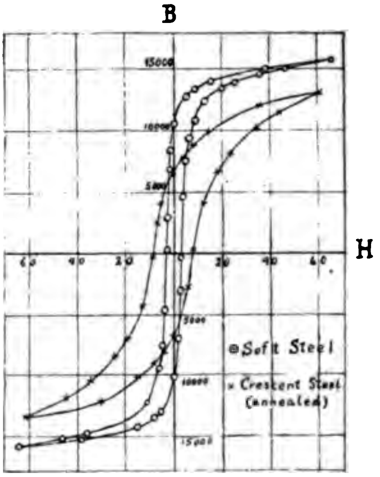

Fig. 86.

On decreasing the current again to zero and increasing it in the positive direction to the original value, the curve will have a form resembling closely the first branch. The closed curve is called a hysteresis curve. The value of B for zero current is termed the "*remanence*" and the value of H for zero B is called the "*coercive force*" of the iron.

The arrangement of the apparatus is the same as in the preceding exercise except that a commutator is inserted in the secondary circuit so that the throw of the galvanometer may be always in one direction. For greater

accuracy and higher sensitiveness it is better to increase the number of turns in the secondary coil. Start with a value of H corresponding to about 6 amperes. Decrease i by large steps until the current is zero. Commute the current and increase it by rather small steps, the variations of B being large on this part of the curve, until the original value of i is obtained. Now reverse the commutator in the secondary circuit and repeat the decrease, reversal and increase of the current.

The deflections of the galvanometer are taken always in the same direction in order to eliminate any possible errors arising from an asymmetrical placing of the scale. It is apparent that the sum of the deflections corresponding to the two branches of the curve must be equal. Let the total sum of each branch be m, then the total deflection Σd, corresponding to the highest point of the curve is m/2, and this is to be substituted in Equation 128, similarly for all other points of the curve. This enables us to locate the origin of the coordinates. Plot B and H.

FORM OF RECORD.

Exercise 66. Hysteresis curve for iron or steel.

Date.............................

	i	H	d	Σd	B
Record apparatus as in preceding exercise.					

TABLES.

TABLE 1.—Atomic Weights of Some Elements

Oxygen - - 16.0.

Aluminium	27.1	Nitrogen	14.04
Cadmium	112.3	Platinum	195.0
Chlorine	35.45	Potassium	39.14
Copper	63.6	Silver	107.93
Gold	197.3	Sodium	23 05
Lead	206.9	Sulphur	32.06
Mercury	200.0	Tin	119.0
Nickel	58.7	Zink	65.40

TABLE 2.—Densities of Water at Different Temperatures.

t.	d.	t.	d.	t.	d.
0°	0.99988	11	0.99965	21	0 99806
1	0.99993	12	0.99955	22	0.99784
2	0.99997	13	0.99943	23	0.99761
3	0.99999	14	0.99930	24	0 99738
4	1.00000	15	0.99915	25	0.99713
5	0.99999	16	0.99900	26	0.99688
6	0 99997	17	0.99884	27	0.99661
7	0.99993	18	0.99866	28	0.99634
8	0.99988	19	0.99847	29	0.99606
9	0.99982	20	0.99827	30	0.99577
10	0.99974				

TABLE III.—DENSITIES OF VARIOUS BODIES.

Alcohol at 20°C	0.789	Iron, wrought	7.86
Aluminium	2.58	Lead	11.3
Brass (about)	8.5	Mercury at 0°C	13.596
Brick	2.1	Nickel	8.9
Copper	8.92	Oak	0.8
Cork	0.24	Paraffin	0.90
Diamond	3.52	Pine	0.5
Glass, common	2.6	Platinum	21.50
" heavy flint	3.7	Quartz	2.65
Gold	19.3	Silver	10.53
Ice at 0°C	0.91	Tin	7.29
Iron, cast	7.4	Zinc	7.15

TABLE IV.—COEFFICIENTS OF ELASTICITY.

Substance.	Volume Elasticity. e.	Simple Rigidity. n.	Young's Modulus. M.
Distilled water	.222 . 10^{11}		
Glass (flint)	3.47 to 4.15 "	2.35 to 2.40 . 10^{11}	5.74 to 6.03 . 10^{11}
Brass	10.02 " 10.85 "	3.44 to 4.03 "	9.48 to 11.2 "
Steel	18.41 "	8.19 "	20.2 to 24.5 "
Iron (wrought)	14.56 "	7.69 "	19.63 "
Iron (cast)	9.64 "	5.32 "	13.49 "
Copper	16.84 "	4.40 to 4.47 "	11.72 to 12.34 "

TABLE V.—MOMENTS OF INERTIA.

Uniform thin Rod, axis through middle, length $= l$ . . . . $I = M\dfrac{l^2}{12}$

Rectangular Lamina, axis through center and *parallel* to one side, a and b length of sides, a the side bisected . . . $I = M\dfrac{a^2}{12}$

Rectangular Lamina, axis through center and *perpendicular* to the plane, a and b length of sides . . . . $I = M\dfrac{a^2+b^2}{12}$

Rectangular Parallelopiped, axis through center and *perpendicular* to a side; a, b and c length of sides, axis perpendicular to side contained by a and b . . . $I = M\dfrac{a^2+b^2}{12}$

MOMENTS OF INERTIA.—(CONTINUED).

Circular Plate, axis through center perpendicular to the plate, radius $= r$.	$I = M\frac{r^2}{2}$
Circular Ring, axis through center perpendicular to plane of ring, outer radius $=$ R, inner radius $= r$	$I = M\frac{R^2 + r^2}{2}$
Right Cylinder, axis the axis of figure, $r =$ radius of section	$I = M\frac{r^2}{2}$
Sphere, axis any diameter, $r =$ radius . .	$I = \frac{2}{5}Mr^2$

TABLE VI.—BOILING POINT OF WATER UNDER DIFFERENT BAROMETRIC PRESSURES.

Barometric press.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
72.	98.50	98.54	98.58	98.61	98.65	98.69	98.73	98.77	98.80	98.84
73.	98.88	98.92	98.96	98.99	99.03	99.07	99.11	99.14	99.18	99.22
74.	99.26	99.30	99.33	99.37	99.41	99.44	99.48	99.52	99.56	99.59
75.	99.63	99.67	99.71	99.74	99.78	99.82	99.85	99.89	99.93	99.96
76.	100.00	100.04	100.07	100.11	100.15	100.18	100.22	100.26	100.29	100.33
77.	100.36	100.40	100.44	100.47	100.51	100.55	100.58	100.62	100.65	100.69

TABLE VII.—SPECIFIC HEATS.

Alcohol	(17°)	0.58	Iron	15-100°	0.113
Mercury	"	0.034	Lead	"	0.032
Turpentine	"	0.43	Nickel	"	0.11
Brass	15-100°	0.094	Platinum	"	0.032
Copper	"	0.094	Quartz	"	0.191
Glass	"	0.19	Silver	"	0.057
Gold	"	0.032	Tin	"	0.056
Ice	"	0.504	Zinc	"	0.094

TABLE VIII.—Electrical Resistance of Metals.

(a). Specific conductivity, referred to mercury.

Aluminium (soft)	32.35	Nickel (soft)	3.14
Copper (pure)	59	Platinum	14.4
Iron	9.75	Silver (soft)	62.6
Mercury	1	Tin	7

(b.) Resistance, in Ohms at 0°C. of wire 100 cm. long, 1 mm. diameter.

		Rate of Change in Resistance per Degree Centigrade
Aluminium	0.03699	0.00388
Copper	0.02062	0.00388
German-silver	0.2660	0.00044
Iron	0.1234	0.00055
Mercury	1.198	0.00072
Platinum	0.1150	... ..
Silver	0.02019	0.00377

TABLE IX.—Numbers Frequently Required.

1 cm = 0.3937 in.	1 in. = 2.540 cm.
1 mile = 1.61 km.	1 km. = 0.6215 mile.
π = 3.14159.	log π = 0.49715.
π^2 = 9.8696.	log π^2 = 0.99430.

Basis of natural logarithms: e = 2.7183.
log e = 0.43429.

Factor to convert common into Naperian logs....2.3026.

Density of air at 0°C. under a barometric pressure of 76 cms. 0.001293 gm/cm³.
Coefficient of expansion of air . . 0.00367.
Velocity of sound in air at 0°C. . 332.4 m/sec.
1 calorie = 4.19 × 10^7 ergs for water at 15°C.
Heat of fusion of water . . . . 80 cal.
Heat of vaporization of water . 537 cal.
1 atmo. pressure = 1.0132 × 10^6 dynes/cm².
g at latitude 45° and sea level = 980.63 cm/sec².

n	πn	$\frac{1}{4}\pi n^2$	n^2	n^3	$\sqrt{n}$
1	3,1416	0,7854	1	1	1,0000
2	6.2832	3.1416	4	8	4142
3	9,4248	7,0686	9	27	7321
4	12.566	12.566	16	64	2,0000
5	15.708	19.635	25	125	2361
6	18.850	28.274	36	216	4495
7	21.991	38.485	49	343	6458
8	25.133	50.265	64	512	8284
9	28.274	63.617	81	729	3.0000
10	31.416	78.540	100	1000	1623
11	34.557	95.03	121	1331	3166
12	37.699	113.10	144	1728	4641
13	40.841	132.73	169	2197	6056
14	43.982	153.94	196	2744	7417
15	47.124	176.17	225	3375	8730
16	50.265	201.06	256	4096	4.0000
17	53.407	226.98	289	4913	1231
18	56.549	254.47	324	5832	2426
19	59.690	283.53	361	6859	3589
20	62.832	314.16	400	8000	4721
21	65.973	346.36	441	9261	5826
22	69.115	380.13	484	10648	6904
23	72.257	415.48	529	12167	7958
24	75.398	452.39	576	13824	8990
25	78.540	490.87	625	15625	5.0000
26	81.68	530.93	676	17576	099
27	84.82	572.55	729	19683	196
28	87.96	615.75	784	21952	291
29	91.11	660.52	841	24389	385
30	94.25	706.86	900	27000	477
31	97.39	754.77	961	29791	568
32	100.53	804.25	1024	32768	657
33	103.67	855.30	1089	35937	745
34	106.81	907.92	1156	39304	831
35	109.96	962.11	1225	42875	916
36	113.10	1017.9	1296	46656	6.000
37	116.24	1075.2	1369	50653	083
38	119.38	1134.1	1444	54872	164
39	122.52	1194.6	1521	59319	245
40	125.66	1256.6	1600	64000	325
41	128.81	1320.3	1681	68921	403
42	131.95	1385.4	1764	74088	481
43	135.09	1452.2	1849	79507	557
44	138.23	1520.5	1936	85184	633
45	141.37	1590.4	2025	91125	708
46	144.51	1661.9	2116	97336	782
47	147.65	1734.9	2209	103823	856
48	150.80	1809.6	2304	110592	928
49	153.94	1885.7	2401	117649	7,000
50	157,08	1963,5	2500	125000	071

n	πn	$\frac{1}{4}\pi n^2$	n^2	n^3	$\sqrt{n}$
51	160,22	2042,8	2601	132651	7.141
52	163.36	2123.7	2704	140608	211
53	166.50	2206,2	2809	148877	280
54	169,65	2290,2	2916	157464	348
55	172.79	2375,8	3025	166375	416
56	175.93	2463,0	3136	175616	483
57	179,07	2551,8	3249	185193	550
58	182.21	2642,1	3364	195112	616
59	185,35	2734,0	3481	205379	681
60	188,50	2827,4	3600	216000	746
61	191.64	2922,5	3721	226981	810
62	194.78	3019.1	3844	238328	874
63	197,92	3117,2	3969	250047	937
64	201,06	3217,0	4096	262144	8,000
65	204,20	3318,3	4225	274625	062
66	207,35	3421,2	4356	287496	124
67	210,49	3525,7	4489	300763	185
68	213,63	3631,7	4624	314432	246
69	216,77	3739.3	4761	328509	307
70	219,91	3848,5	4900	343000	367
71	223,05	3959,2	5041	357911	426
72	226,19	4071,5	5184	373248	485
73	229,34	41854.	5329	389017	544
74	232,48	4300,8	5476	405224	602
75	235,62	4417,9	5625	421875	660
76	238,76	4536,5	5776	438976	718
77	241,90	4656,6	5929	456533	775
78	245,04	4778,4	6084	474552	832
79	248,19	4901,7	6241	493039	888
80	251,33	5026,6	6400	512000	944
81	254,47	5153,0	6561	531441	9,000
82	257,61	5281,0	6724	551368	055
83	260,75	5410,6	6889	571787	110
84	263,89	5541,8	7056	592704	165
85	267,04	5674,5	7225	614125	219
86	270,18	5808,8	7396	636056	274
87	273,32	5944,7	7569	658503	327
88	276,46	6082,1	7744	681472	381
89	279,60	6221,1	7921	704969	434
90	282,74	6361,7	8100	729000	487
91	285,88	6503,9	8281	753571	539
92	289,03	6647,6	8464	778688	592
93	292,17	6792,9	8649	804357	644
94	295,31	6939,8	8836	830584	695
95	298,45	7088,2	9025	857375	747
96	301,59	7238,2	9216	884736	798
97	304,73	7389,8	9409	912673	849
98	307,88	7543,0	9604	941192	899
99	311,02	7697,7	9801	970299	950
100	314,16	7854,0	10000	1000000	10,000

Trigonometric Functions

	Arc	Sine	Tangent	log arc	log sin	log tan
1°	0.0175	0.0175	0.0175	$\bar{2}$,2419	$\bar{2}$,2419	$\bar{2}$,2419
2	0349	0349	0349	5428	5428	5431
3	0524	0523	0524	7190	7188	7194
4	0698	0698	0699	8439	8436	8446
5	0873	0872	0875	9408	9403	9420
6	1047	1045	1051	$\bar{1}$,0200	$\bar{1}$,0192	$\bar{1}$,0210
7	1222	1219	1228	0870	0859	0891
8	1396	1392	1405	1450	1436	1478
9	1571	1564	1584	1961	1943	1997
10	1745	1736	1763	2419	2397	2463
11	1920	1908	1944	2833	2806	2887
12	2094	2079	2126	3210	3179	3275
13	2269	2250	2309	3558	3521	3634
14	2443	2419	2493	3879	3837	3968
15	2618	2588	2679	4180	4130	4281
16	2793	2756	2867	4461	4403	4575
17	2967	2924	3057	4723	4659	4853
18	3142	3090	3249	4972	4900	5118
19	3316	3256	3443	5206	5126	5370
20	3491	3420	3640	5429	5341	5611
21	3665	3584	3839	5641	5543	5842
22	3840	3746	4040	5843	5736	6064
23	4014	3907	4245	6036	5919	6279
24	4189	4067	4452	6221	6093	6486
25	4363	4226	4663	6398	6259	6687
26	4538	4384	4877	6569	6418	6882
27	4712	4540	5095	6732	6570	7072
28	4887	4695	5317	6890	6716	7257
29	5061	4848	5543	7042	6856	7438
30	5236	5000	5774	7190	6990	7614
31	5411	5150	6009	7333	7118	7788
32	5585	5299	6249	7470	7242	7958
33	5760	5446	6494	7604	7361	8125
34	5934	5592	6745	7733	7476	8290
35	6109	5736	7002	7860	7586	8452
36	6283	5878	7265	7982	7692	8613
37	6458	6018	7536	8101	7795	8771
38	6632	6157	7813	8216	7893	8928
39	6807	6293	8098	8330	7989	9084
40	6981	6428	8391	8439	8081	9238
41	7156	6561	8693	8546	8169	9392
42	7330	6691	9004	8651	8255	9544
43	7505	6820	9325	8753	8338	9697
44	7679	6947	9657	8853	8418	9848
45	7854	7071	1.0000	8951	8495	0.0000

Trigonometric Functions

	Arc	Sine	Tangent	log arc	log sin	log tan
46°	0,8029	0.7193	1.0355	$\bar{1}$,9047	$\bar{1}$.8569	0,0152
47	8203	7314	0724	9140	8641	0303
48	8378	7431	1106	9231	8711	0456
49	8552	7547	1504	9321	8778	0608
50	8727	7660	1918	9409	8843	0762
51	8901	7771	2349	9494	8905	0916
52	9076	7880	2799	9579	8965	1072
53	9250	7986	3270	9661	9023	1229
54	9425	8090	3764	9743	9080	1387
55	9599	8192	4281	9822	9134	1548
56	9774	8290	4826	9901	9186	1710
57	9948	8387	5399	9977	9236	1875
58	1.0123	8480	6003	0,0053	9284	2042
59	0297	8572	6643	0127	9331	2212
60	0472	8660	7321	0200	9375	2386
61	0647	8746	8040	0272	9418	2562
62	0821	8829	8807	0343	9459	2743
63	0996	8910	9626	0412	9499	2928
64	1170	8988	2,0503	0480	9537	3118
65	1345	9063	1445	0548	9573	3313
66	1519	9135	2460	0614	9607	3514
67	1694	9205	3559	0680	9640	3721
68	1868	9272	4751	0744	9672	3936
69	2043	9336	6051	0807	9702	4158
70	2217	9397	7475	0870	9730	4389
71	2392	9455	9042	0931	9757	4630
72	2566	9511	3.0777	0992	9782	4882
73	2741	9563	2709	1052	9806	5147
74	2915	9613	4874	1111	9828	5425
75	3090	9659	7321	1169	9849	5719
76	3265	9703	4.0108	1227	9869	6032
77	3439	9744	3315	1284	9887	6366
78	3614	9781	7046	1340	9904	6725
79	3788	9816	5.1446	1395	9919	7113
80	3963	9848	6713	1450	9934	7537
81	4137	9877	6.3138	1504	9946	8003
82	4312	9903	7.1154	1557	9958	8522
83	4486	9925	8.1443	1609	9968	9109
84	4661	9945	9.5144	1662	9976	9784
85	4835	9962	11.4301	1713	9983	1.0580
86	5010	9976	14.3007	1764	9989	1554
87	5184	9986	19.0811	1814	9994	2806
88	5359	9994	28.6363	1864	9997	4569
89	5533	9998	57.2900	1913	9999	7581
90	5708	1.0000	∞	1961	0.0000	∞

Logarithms

N	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774

LOGARITHMS

N	0	1	2	3	4	5	6	7	8	9
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996
100	00000	0043	0087	0130	0173	0217	0260	0303	0346	0389
101	00432	0475	0518	0561	0604	0647	0689	0732	0775	0817
102	00860	0903	0945	0988	1030	1072	1115	1157	1199	1242
103	01284	1326	1368	1410	1452	1494	1536	1578	1620	1662
104	01703	1745	1787	1828	1870	1912	1953	1995	2036	2078
105	02119	2160	2202	2243	2284	2325	2366	2407	2449	2490
106	02531	2572	2612	2653	2694	2735	2776	2816	2857	2898
107	02938	2979	3019	3060	3100	3141	3181	3222	3262	3302
108	03342	3383	3423	3463	3503	3543	3583	3623	3663	3703
109	03743	3782	3822	3862	3902	3941	3981	4021	4060	4100
110	04139	4179	4218	4258	4297	4336	4376	4415	4454	4493

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The first part of this book, as indicated by the title, consists of Laboratory Notes for a Beginner's Course in Quantitative Analysis. It considers the subjects of Gravimetric and Volumetric Analysis, for beginners, by means of the chemical analysis of a set of substances properly numbered, in each case giving the methods to be followed in such analysis; also the methods for calculating and preparing volumetric standard solutions, generally following the course offered by Professor Cheever to his students. It also considers the methods for the determination of the specific gravities of various liquids and solids.

Although a number of the analyses contained in Part I. may be of only approximate accuracy, and of small commercial value, such are yet included with a special purpose, to wit:—that they may supply the student with a wider range of work and a greater diversity of chemical manipulation. This was Professor Cheever's idea, and it is certainly a good one, especially since, in most cases, the work of the beginner simply serves to emphasize the necessity of careful scrutiny of details and methods for practical work in the future.

Part I. is offered, then, for the use of schools and colleges, and it is intended to supply a source of elementary information upon the subject of Quantitative Chemical Analysis rarely offered in such form in works upon that subject.—Preface

The author was for many years Professor of Metallurgy in the University of Michigan, and the methods here presented are those mostly offered by him to his students. As a beginner's book in quantitative analysis, it will be found eminently practical, and it can be honestly recommended to the student who desires a source of elementary information upon this branch of applied science The book is divided into two parts, the first consisting of laboratory notes for beginners. The subjects of gravimetric and volumetric analysis are considered by means of the chemical analysis of a set of substances, properly numbered, in each case giving the methods to be followed in such analysis, and also the methods of calculating and preparing volumetric standard solutions, etc. Methods for the determination of specific gravities of various liquids and solids are also considered.

Part II. contains a number of select methods in inorganic quantitative analysis, such as the analysis of limestone, iron ores, manganese ores, steel, the analysis of coal, water, mineral phosphates, smelting ores, lead slags, copper, arsenic, bismuth, etc. A chapter on reagents concludes the work.—*Pharmaceutical Era.*

D'OOGE.—*Helps to the Study of Classical Mythology; for the Lower Grades and Secondary Schools.* By B. L. D'Ooge, Professor in the Michigan State Normal College. 12 mo. 180 pages. Cloth. 45 cents.

A bibliography based on practical experience. The author is a professor in the Michigan State Normal College. As the myths of all nations manifest themselves first in religion, secondly in art, and third in literature, these reading references are grouped in the above classes. One section is devoted to the study of mythology in the grades, and an introductory chapter gives hints for teaching the subject in the lower grades. The books suggested in the body of the work are given in one alphabet at the end, with publishers and prices; there are also blank pages for additional references, and a good general index.—*Publishers Weekly.*

DOW.—*Outlines and References in European History.* For the use especially of Students in History 1 and 2, University of Michigan. Part I. From the Fourth to the Ninth Century. By Earle Wilbur Dow, University of Michigan. 42 pages. Pamphlet. 35 cents.

DOW.—*Outlines and References in European History.* For the use especially of Students in History, 1 and 2, University of Michigan. Part II. From the Ninth to the Thirteenth Century. By Earle Wilbur Dow, University of Michigan. 66 pages. Pamphlet. 35 cents.

DWYER.—*Cases on Private International Law.* By John W. Dwyer, University of Michigan. 8vo. 509 pages. Buckram, $2.50.

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The cases are not new, many of them are quite old, but are well chosen with the view of illustrating international law where the contests arise between parties, one of whom is domiciled in this country, and the other in a foreign country, or between parties residing in different states in this country. These cases, which have been selected by the author with much good judgment, illustrate with great fullness under the conditions above stated the law pertaining to marriage, divorce, legitimacy, guardians, administration, judgments, corporations, unmovables, movables, attachment, contracts, statute of frauds, torts, procedure. Also domicil of students, sailors, apprentices, insane persons, infants, married women, commercial domicil, reverter, domicil in uncivilized countries, domicil of origin and choice. These cases may well be called leading cases, and will afford much aid to the seeker of information analogous to the subjects in these cases discussed.—*Central Law Journal.*

I have examined with care the copy of Prof. Dwyer's selected cases on Private International Law, which you sent me some weeks since, and I have no hesitation in saying that it is the best selection I have yet seen of cases upon this subject. It is especially satisfactory on the subject of "Domicile", "Administration" and "Guardianship".

Had I not already made a selection of cases for the use of my classes in the Law School of the University of Maryland, I should unhesitatingly select this book as a text-book for that subject. As it is I find that I am using several of the cases selected by Mr. Dwyer, and have been for some time.

HENRY STOCKBRIDGE,
Baltimore, Md.

I have received and examined with much fullness and interest the volume of "Cases on Private International Law", by Dwyer. I am much pleased with the selection of cases, and think the topics covered by the cases are those of most interest and importance in connection with the subject of Private International Law. I have taught Private International Law by means of lectures for several

years, and feel quite competent to judge of the sagacity in the selection of the cases, and of the admirable arrangement of them in their sequential order by Mr. Dwyer in his volume of cases. I can most cordially commend the volume, and shall myself hereafter use it in the Iowa College of Law. C. C. COLE,

Iowa College of Law.

I have examined with care Dwyer's "Cases on Private International Law", and find them judiciously selected and edited. The author has succeeded in presenting in a small compass many important decisions in which the leading doctrines of this branch of the law are exhibited and applied in a manner to make the book useful alike to the student and to the practitioner. I trust the work will meet with the favorable reception it deserves. GEO. B. YOUNG,

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